Effect of Soliton Interaction on Timing Jitter in Communication Systems

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Abstract— Timing jitter in soliton communication systems is studied, taking into account both soliton interaction and amplifier noise. Deviations from Gordon–Haus jitter for closely spaced solitons are observed. A new analytical model for the timing jitter is proposed. The model presented considers interaction in a random sequence of solitons and the effect of the amplified spontaneous emission noise added in each amplification stage. We obtain a good agreement between the new analytical model and simulation results for practical communication systems.

Index Terms—Communication systems, solitons, timing jitter.

I. INTRODUCTION

IN a long-haul soliton communication system, lumped amplifiers are used to compensate for fiber losses. The noise added to the signal by each amplifier induces an uncertainty in the soliton arrival time called jitter. Gordon and Haus showed that the statistics of the jitter due to the spontaneous emission noise added by lumped amplifiers is Gaussian, with a variance proportional to the cube of the total distance of the link [1].

Recent experiments have shown significant deviations from the Gaussian distribution [2]. The study of the physical effects leading to this deviation has been the subject of considerable amount of study. It was pointed out that soliton interaction, acoustic effects, and polarization mode dispersion can lead to deviations from the Gordon–Haus result [3], [4]. However, the soliton interaction is likely to have the dominant effect for high bit rate systems.

In this work we derive a model for the soliton arrival time statistics considering the soliton interaction and the amplified spontaneous emission noise. As full Monte Carlo simulations for arrival time calculation require huge amount of computing time, this new model can be useful in system design [5], [6]. We validate this new model with numerical simulations of practical communication systems, working at 10, 20, and 40 Gb/s. The results show that soliton interaction produces a considerable change in the jitter statistics.

II. DEVIATION FROM THE GAUSSIAN STATISTICS (SIMULATION RESULT)

Gordon–Haus jitter due to the spontaneous emission noise

TABLE I

Bit rate	20 Gbit/s
Pulse width	10.0 ps
Wavelength	1.55 μm
Total length	3 000 km
Span between amplifiers	20 km
Dispersion	0.1 ps/(nm.km)
Attenuation	0.1 dB/km
Spontaneous emission factor	2.0
Nonlinear coefficient	$3.7\ \mathrm{W}^{-1}.\mathrm{km}^{-1}$

added in each amplifier is described by a Gaussian probability density function (pdf) with the standard deviation given by [1]

$$\sigma = \left[\frac{n_{\mathrm{sp}} n_2 D h (G-1) L^3}{9 T_o A_{\mathrm{eff}} L_{\mathrm{amp}} Q} \right]^{1/2} \tag{1}$$

where n_{sp} is the spontaneous emission factor, n_2 is the nonlinear parameter of the fiber, D is the first-order group-velocity dispersion factor of the fiber, h is Planck's constant, G is the amplifier gain, L is the total length of the link, T_o is the soliton width, A_{eff} is the effective mode area, L_{amp} is the amplifier spacing, and Q is the power-enhancement factor [7].

In communication systems, the solitons are launched close to each other, in order to achieve high-bit rates, which leads to mutual interactions. This perturbation of the soliton propagation due to neighbor pulses significantly changes the timing jitter statistics.

To study the effect of the soliton interaction we have solved the nonlinear Schrödinger equation (NLSE) for a pseudorandom sequence of solitons. We considered an optical communication system operating at 20 Gb/s, with the parameters shown in Table I.

The separation between amplifiers was kept much less than the dispersion length, in order to avoid the dispersive waves that could become another source of timing jitter. We have solved numerically the NLSE equation for 1600 bits from a pseudo-random sequence, using 64 sample points for each bit slot for accurate jitter measurements. Fig. 1 shows the pdf of the arrival time obtained by simulation and the Gordon–Haus result.

We clearly see the deviation from the Gordon–Haus result due to the soliton interaction. The pdf of the arrival time is not Gaussian and it is broader, so we should expect a degradation of the system bit error rate due to soliton interaction.
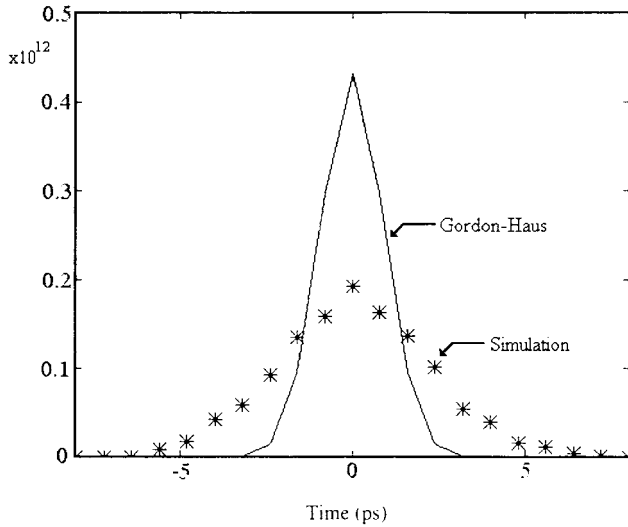
Fig. 1. Probability density function for the timing jitter of a 20 Gb/s system. The line is the Gordon–Haus result and the starts are the simulation result.

III. ANALYTICAL MODEL OF SOLITON INTERACTION

When we launch more than one pulse in a fiber, the presence of neighbor pulses disturbs the soliton, which changes its time position. If the neighbor solitons are in phase, this interaction is attractive and solitons move closer.

To study multiple-pulse systems, we start with the NLSE for the case of two solitons. An approximate solution for the case of two solitons based on the quasiparticle approach was presented by Karpman and Solov'ev [8]. Another approximation, was derived by Gordon directly from the exact two solitons equation [9]. The expression for a soliton pair has the general form

$$u(\tau, q) = \exp(i\Omega)\{A_1 \, \text{sech} \, [A_1(\tau - q)] \exp(i\theta_1) \\ + A_2 \, \text{sech} \, [A_2(\tau + q)] \exp(i\theta_2)\}. \tag{2}$$

At each point inside the fiber the normalized separation between the solitons is $2q$, the amplitudes of the two solitons are A_1 and A_2, and the relative phase difference is $\theta_2 - \theta_1$. For simplicity of notation, we introduce $\psi = (\theta_2 - \theta_1)/2$.

It can be shown that

$$\rho \exp(q + i\psi) = 2 \cosh(\zeta_o + i\rho\zeta) \tag{3}$$

where ζ is the distance of propagation, ρ and ζ_o are constants determined by the initial separation, phase difference and its first derivative. The first and second space derivatives of (3) combine to yield the equations of motion [8], [9]

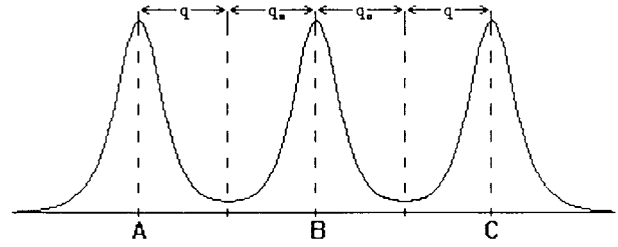$$\frac{\partial^2 q}{\partial \zeta^2} = -4 \exp(-2q) \cos(2\psi) \tag{4}$$

$$\frac{\partial^2 \psi}{\partial \zeta^2} = 4 \exp(-2q) \sin(2\psi). \tag{5}$$

Equations (4) and (5) show that the dynamics of the soliton pair are entirely due to interaction forces that depend exponentially on their separation and sinusoidally on their relative phase. If q_o and ψ_o are the initial separation and phase difference respectively, we obtain the followings expressions for q and ψ during the propagation in (6) and (7) shown at the bottom of the page.

Since a zero phase difference between neighboring solitons leads to the worst case, we consider this case only. Choosing $\psi_o = 0$ in (6), we obtain the following expression for the separation between the two solitons:

$$q = q_o + \ln[\cos(a\zeta)] \tag{8}$$

where $a = 2\exp(-q_o)$.

To extend the above results to the three-soliton case, we note that the force between adjacent solitons depends on their separation and relative phases. Then in a system of three solitons where the distance and relative phases between side solitons (Fig. 2, A and C) and the middle soliton B are the same, the interaction of solitons A and C relative to B, are of opposing signs. The middle soliton is fixed because of these balancing forces.

In the two-soliton case, the separation is equal to $2q$. However, in the three-soliton case, because the middle soliton is fixed, the separation between side solitons and the middle one is $q_o + q$ where q_o, is the initial separation. If we use this fact in (4) and (5), we obtain the following pair of equations for the three-soliton dynamics:

$$\frac{\partial^2 q}{\partial \zeta^2} = -2a \exp(-q) \cos(2\psi) \tag{9}$$

$$\frac{\partial^2 \psi}{\partial \zeta^2} = 2a \exp(-q) \sin(2\psi) \tag{10}$$

where $a = 2\exp(-q_o)$.

Fig. 2. In the three-soliton system in phase, the interactions of solitons A and C relative to B are of opposing signs.

$$q(\zeta) = q_o + \frac{1}{2} \ln \left[\frac{\cosh[4\exp(-q_o)\sin(\psi_o)\zeta] + \cos[4\exp(-q_o)\cos(\psi_o)\zeta]}{2} \right] \tag{6}$$

$$\psi(\zeta) = \psi_o + \frac{1}{2i} \ln \left[\frac{\cos[2\exp(-q_o)\exp(-i\psi_o)\zeta]}{\cos[2\exp(-q_o)\exp(i\psi_o)\zeta]} \right]. \tag{7}$$
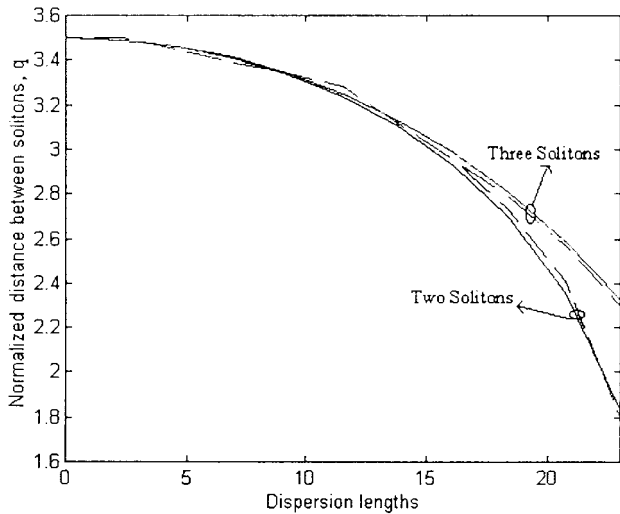
Fig. 3. Normalized distance between solitons. The solid line is the analytical solution, the dashed line is obtained by numerical solution of the NLSE.
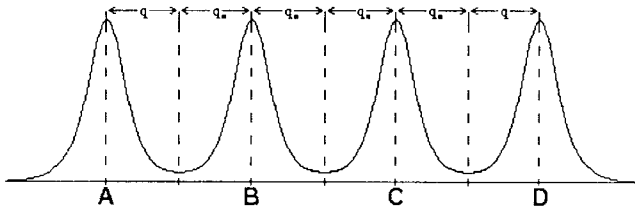

Fig. 4. In the four-soliton system the middle solitons (B and C) are practically fixed because each one is surrounded by neighbor pulses exerting opposing forces.

The solution of (9) and (10), in the case of in-phase pulses provides:

$$q = q_o + \ln\left[\cos^2\left(\frac{1}{\sqrt{2}}a\zeta\right)\right]. \qquad (11)$$

Comparing (11) with the corresponding solution (8) for the two solitons case, we see that despite the square in the cosine function, in the case of three solitons, the period is $\sqrt{2}$ times the period of the two-soliton system. This means that the soliton interaction is weaker for the case of three-soliton compared to the case of two-soliton.

To test the validity of our analytical result, we performed a simulation by numerically solving the NLSE for the case of two and three-soliton. The results are shown in Fig. 3. As we can see, the numerical results are in agreement with the analytical ones.

To generalize our results, we start with the four-soliton case, in which the two middle solitons (B and C in Fig. 4) are practically fixed, because each one is surrounded by neighbor pulses exerting opposing forces. With a good degree of approximation, we can assume that the two middle solitons are fixed and the side solitons (A and D) behave in a similar way to the three-soliton system.

It is obvious that we can extend this approximation to the case of more than four solitons. In a long sequence of pulses, the only ones where the interaction forces are important are the first and the last one of the sequence. So the case of more than three solitons can be reduced to the three-soliton system.
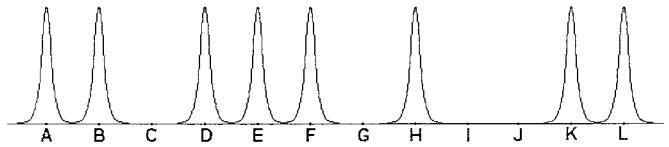

Fig. 5. To analyze the solitons interaction in a long sequence of pulses we can divide the sequence in small slices and analyze each slice.

In a lightwave communication system making use of pulse-code modulation, the pulse sequence is random, containing long sequences of pulses, but also isolated and pairs of pulses. To analyze the solitons interaction in a long sequence of pulses, we can divide the sequence in small slices and reduce each slice to one of the cases presented above.

In Fig. 5, we have a sequence of 12 bits that we can reduce to the cases of isolated pulse (H), two-soliton case (A and B, K, and L) and three-soliton system (D and E and F).

In order to analytically determine the arrival time statistics, it is enough to consider at most four neighbor solitons because the interaction forces decrease exponentially with solitons separation. We consider a random binary sequence with $2^5 - 1$ bits that contains all possible sequences of 5 bits [10].

It can be shown that the statistics of the arrival times is represented by a bar graph, with 50% weight for zero-time deviation, 25% corresponding to a deviation equal to the two-soliton system, and 25% with a deviation equal of the three-soliton system. Moreover, since the deviation can be positive or negative the 25% probability of two and three solitons should be split into two bars of 12.5% on the two sides. So, we should expect a jitter pdf given by

$$p(t) = \tfrac{1}{2}\delta(t) + \tfrac{1}{8}\delta(t - t_2) + \tfrac{1}{8}\delta(t + t_2)$$
$$+ \tfrac{1}{8}\delta(t - t_3) + \tfrac{1}{8}\delta(t + t_3). \qquad (12)$$

Values of t_2 and t_3 are given by

$$t_2 = T_o \ln\left[\cos\left(\frac{a}{L_D}z\right)\right] \qquad (13)$$

$$t_3 = T_o \ln\left[\cos^2\left(\frac{a}{\sqrt{2}L_D}z\right)\right] \qquad (14)$$

where T_o is the soliton width, L_D the dispersion length, $a = 2\exp[-T_{\text{bit}}/(2T_o)]$, T_{bit} is the bit period and z is the total length of the optical communication system.

To check the validity of our analytical result, we repeat the 20 Gb/s system simulation, described in Section II, with an ideal amplifier ($n_{\text{sp}} = 0$) and with a total length of nine dispersion lengths. From Fig. 6 we can see a good agreement between our analytical deduction and the numerical results. There are a broadening in the delta function produced by the finite numerical resolution (resolution = 0.78 ps), but the position of the side bars are exactly the positions predicted by our model and their relative heights match the prediction of (12).

IV. EFFECT OF SPONTANEOUS-EMISSION NOISE

In a long-haul soliton communication system, the fiber losses are compensated by the use of in line lumped amplifiers. These amplifiers restore the signal power but the amplification
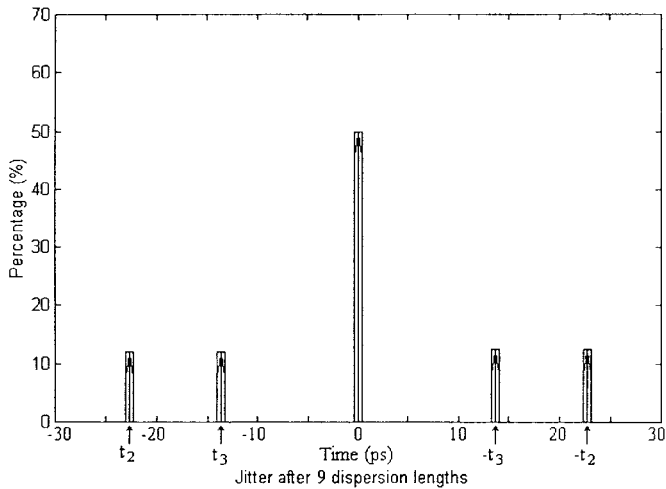
Fig. 6. Arrival time statistics of a pseudorandom sequence, after the propagation of nine dispersion lengths. The bars were obtained by numerical simulation. We used expressions (13) and (14) to determine the t_2 and t_3. The arrows inside the bars are the delta function obtained by expression (12).
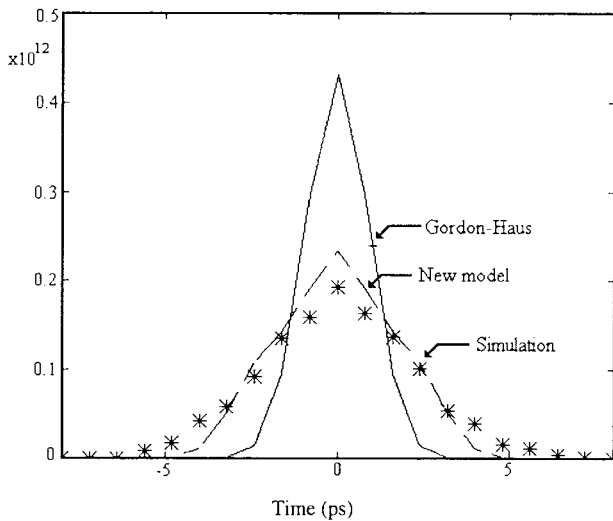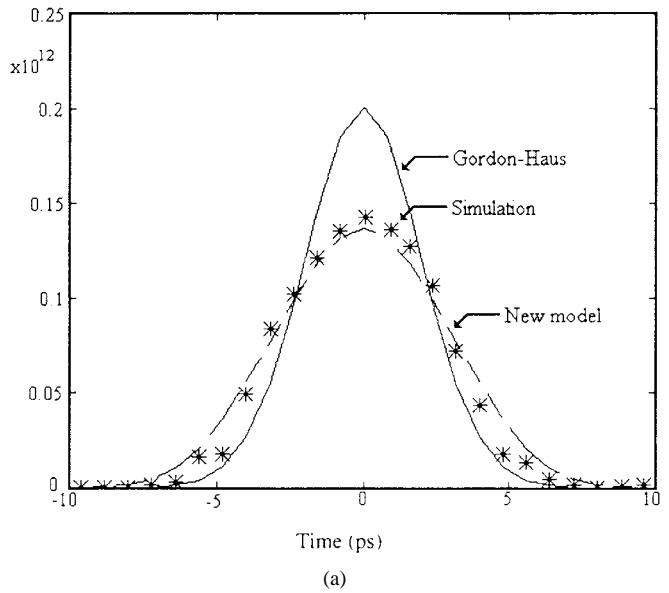

Fig. 7. Probability density function for the 20 Gb/s system. The line is the Gordon–Haus result, the starts are the simulation result, and the dashed-line is our model.

Fig. 8. Probability density function for (a) a 10-Gb/s system and (b) a 40-Gb/s system. The line is the Gordon–Haus result, the starts are the simulation result, and the dashed-line is our model.
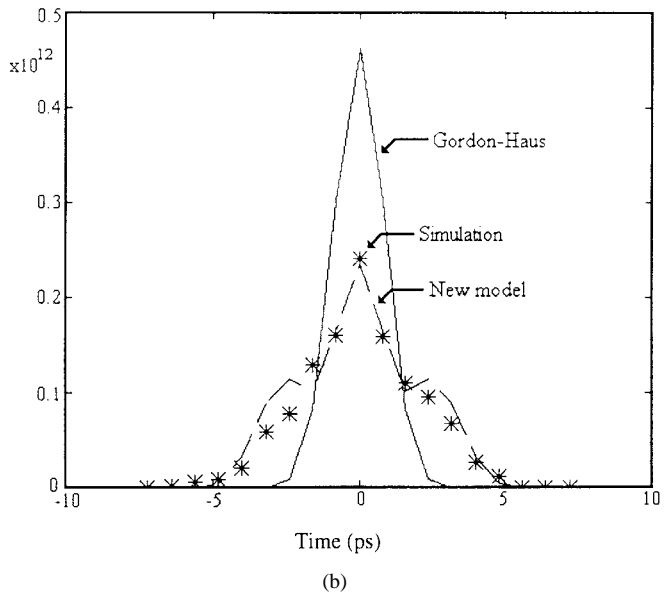
process is accompanied by the emission of spontaneous noise [11]. The noise that is outside the bandwidth of the optical signal can be removed, using an appropriate optical filter, although it is not possible to remove in-band noise. This noise copropagates with the signal.

The copropagation of signal and noise changes the arriving time statistics presented in expression (12) and also degrades the signal noise ratio. Expression (12) is not valid any more, due to the Gordon–Haus effect and because the interaction forces are phase and amplitude sensitive.

In a soliton communication system, the noise power is much smaller than the signal, so we can assume that the noise only produces a small change in the phase and amplitude of each pulse. Making this assumption, we will expect to have a jitter pdf related to the one presented in (12). However, instead of five discrete lines, we should have a five-lobe Gaussian function, with the center of each Gaussian determined by

expressions (13) and (14), and with the variance given by the Gordon–Haus expression (1). So, we can expect to have a jitter pdf given by

$$p(t) = \tfrac{1}{2}f_g(t,\sigma) + \tfrac{1}{8}f_g(t - t_2,\sigma) + \tfrac{1}{8}f_g(t + t_2,\sigma)$$
$$+ \tfrac{1}{8}f_g(t - t_3,\sigma) + \tfrac{1}{8}f_g(t + t_3,\sigma) \qquad (15)$$

where $f_g(t,\sigma)$ is given by

$$f_g(t,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{t^2}{2\sigma^2}\right] \qquad (16)$$

the values of t_2 and t_3 are given by expressions (13) and (14), respectively, and σ is given by (1).

V. COMPARISON BETWEEN THE ANALYTICAL AND NUMERICAL RESULTS

To validate our model, we apply it firstly to the simulation results presented in Fig. 1.

As we can see in Fig. 7, the new model gives a good approximation to the pdf of the arrival time. In particular, the tails of the pdf are very well approximated by our model. Since the tails are the main feature determining error rates values in high quality systems, we can expect improved bit error rates estimation applying this model to practical communication systems.

In order to analyze the behavior of the new model for different bit rates we applied it to systems working at 10 and 40 Gb/s. These are the bit rates of the next generation of optical communication systems. The results are printed in Fig. 8 where we can see again good agreement between the new analytical model and simulation results. Furthermore the deviation of the jitter pdf from the Gaussian shape, is stronger as the bit rate increases.

VI. CONCLUSIONS

We have derived a simple analytical expression for the jitter pdf of soliton systems, taking into account both soliton interaction and amplified spontaneous emission noise. Monte Carlo simulation results have shown good agreement with analytical ones, thus confirming the validity and accuracy of the new model presented. The soliton interaction produces a considerable deviation from jitter Gaussian statistics for high speed systems.

REFERENCES

[1] J. P. Gordon and H. A. Haus, "Random walk of coherently amplified solitons in optical fiber transmission," *Opt. Lett.*, vol. 11, no. 10, pp. 665–667, 1986.
[2] L. F. Mollenauer, P. V. Mamyshev, and M. J. Neubelt, "Measurement of timing jitter in filter-guided soliton transmission at 10 Gbits/s and achievement of 375 Gbits/s-Mm, error free, at 12.5 and 15 Gbits/s," *Opt. Lett.*, vol. 19, no. 10, pp. 704–706, 1994.
[3] C. R. Menyuk, "Non-Gaussian corrections to the Gordon-Haus distribution resulting from soliton interactions," *Opt. Lett.*, vol. 20, no. 3, pp. 285–287, 1995.
[4] T. Georges, "Bit error rate degradation of interacting solitons owing to non-Gaussian statistics," *Electron. Lett.*, vol. 31, no. 14, pp. 1174–1175, 1995.
[5] R. F. S. Ribeiro and Jose R. Ferreira da Rocha, "Computationally efficient methods for bit-error rate evaluation in weakly coherent optical systems," *J. Lightwave Technol.*, vol. 12, pp. 1842–1848, Oct. 1994.
[6] J. R. F. da Rocha, L. B. Ribeiro, and A. N. Pinto, "Semi-analytical method for performance analysis of soliton systems," in *Proc. CLEO/Pacific Rim'95*, Japan, 1995, pp. 43–44.
[7] D. Marcuse, " Simulations to demonstrate reduction of the Gordon-Haus effect," *Opt. Lett.*, vol. 17, no. 1, pp. 34–36, 1992.
[8] V. I. Karpman, and V. V. Solov'ev, " A perturbational approach to the two-soliton systems," *Physica 3D*, 1981, pp. 487–502.
[9] J. P. Gordon, "Interaction forces among solitons in optical fibers," *Opt. Lett.*, vol. 8, no. 11, pp. 596–598, 1983.
[10] M. C. Jeruchim, P. Balaban, and K. S. Shanmugan, *Simulation of Communication Systems*. New York: Plenum, 1992, pp. 284–287.
[11] G. P. Agrawal, *Fiber-Optic Communication Systems*. New York: Wiley, 1992, pp. 329–383.

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