Spatiotemporal coupling in dispersive nonlinear planar waveguides

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The multidimensional nonlinear Schrödinger equation governs the spatial and temporal evolution of an optical field inside a nonlinear dispersive medium. Although spatial (diffractive) and temporal (dispersive) effects can be studied independently in a linear medium, they become mutually coupled in a nonlinear medium. We present the results of numerical simulations showing this spatiotemporal coupling for ultrashort pulses propagating in dispersive Kerr media. We investigate how spatiotemporal coupling affects the behavior of the optical field in each of the four regimes defined by the type of group-velocity dispersion (normal or anomalous) and the type of nonlinearity (focusing or defocusing). We show that dispersion, through spatiotemporal coupling, can either enhance or suppress self-focusing and self-defocusing. Similarly, we demonstrate that diffraction can either enhance or suppress pulse compression or broadening. We also discuss how these effects can be controlled with optical phase modulation, such as that provided by a lens (spatial phase modulation) or frequency chirping (temporal phase modulation). © 1995 Optical Society of America

1. INTRODUCTION

For many years, optical pulse propagation in fibers has been a subject of intense investigation.[1] Apart from the obvious communication applications, optical fibers are important because they provide a simplified environment for studying nonlinear optical effects. Two important consequences of the wave-guiding nature of fibers are (i) that diffractive effects can be eliminated from consideration and, perhaps more importantly, (ii) that nearly constant pulse energies can be maintained over relatively long propagation distances because of relatively low loss in fibers. Thus fibers are excellent tools for studying the interplay of chromatic dispersion and optical nonlinearities, in particular the intensity-dependent refractive index or the Kerr nonlinearity.[1] Planar waveguides are similar to fibers in that there is still good field confinement. But, because the confinement is only one dimensional, it is possible to study the influence of diffractive effects on the field dynamics in the presence of the Kerr nonlinearity responsible for self-focusing or self-defocusing.[2,3] This interplay is particularly interesting when the optical field is in the form of an ultrashort pulse, in which case not only do the dispersive and diffractive effects occur simultaneously but also the spatial behavior is coupled through the nonlinearity to the temporal behavior.[4–8] This nonlinear spatiotemporal coupling has been shown to cause a localized pulse compression in the normal-dispersion regime of self-focusing media[4] and is behind the mechanism by which ultrashort pulses alter the shape of the beam in a defocusing medium.[5] The spatiotemporal coupling of pulses in nonlinear media also plays an important role in Kerr-lens mode locking,[6] in which the self-focusing enhancement in the laser rod for the pulsed field in combination with an effective aperture makes the mode-locked state more stable than continuous-wave operation. The effectiveness of this technique makes it possible to use an intracavity nonlinear element instead of the laser rod itself to induce self-mode locking.[7,8]

In recent years there has been some discussion of the most effective way to model pulse evolution in nonlinear optical media. A full-wave vector Maxwell equation approach[9] is accurate but is also the most computationally intensive. The multidimensional nonlinear Schrödinger equation (NSE) has been a useful tool for the study of pulse evolution in fibers and waveguides for the past three decades.[1] Recently, modifications to the NSE have been suggested as means of compensating for discrepancies caused by the breakdown of the slowly varying amplitude approximation during the self-focusing of ultrashort pulses.[10,11] New beam-propagation algorithms have also been introduced by studying nonparaxial field evolution.[12] We limit the scope of our discussion to paraxial, slowly varying fields and therefore employ a standard split-step Fourier algorithm[1] used for the NSE. We also limit our investigations to planar waveguides and the range of nonlinearities over which the waveguide approximation remains valid.[2,13]

This paper is devoted to studying the effects of spatiotemporal coupling on the optical field behavior in dispersive nonlinear waveguides. In Section 2 we review the multidimensional NSE and then in Section 3 discuss the special cases under which it supports solitons. Sections 4 and 5 present the results of numerical simulations for self-focusing and self-defocusing media, respectively, showing how dispersion can alter the spatial profile of pulsed beam and under which circumstances diffraction can affect temporal pulse shape. We also show that, because of the spatiotemporal coupling implied by the multidimensional NSE, one can control the pulse width in time with manipulation of the optical phase in space, and, conversely, one can control the beam width in space by manipulating the optical phase in time.

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2. NONLINEAR SCHRÖDINGER EQUATION

The numerical simulations presented here are based on the multidimensional NSE.[1] The NSE is derived from Maxwell's equations for the case of an intensity-dependent (Kerr-type) index of refraction of the form $n(\omega) = n_0(\omega) + n_2 I$ (resulting in a cubic nonlinearity in the field dependence of the polarization), where the frequency dependence of the linear index $n(\omega)$ results from chromatic dispersion. We model pulse propagation with the NSE in the two-dimensional (one space and one time dimension) or waveguide approximation, which consists of assuming that diffraction occurs in only one transverse direction, the field behavior in the other direction being determined by the structure of the waveguide. This approximation also holds for the propagation of highly elliptical beams in bulk nonlinear media, where diffraction in one dimension occurs over a much larger distance scale than in the other.

To simplify the model and broaden the applicability of the results, we normalize all variables including the field, which is normalized so that its peak input value is unity. The coordinates are normalized as follows: the transverse spatial coordinate $\xi = x/\sigma$ is normalized to the input beam width σ, the temporal coordinate $\tau = [t - (z/v_g)]/T_0$ is the reduced time normalized to the incident pulse width T_0, and the propagation distance $\zeta = z/L_d$ is measured in units of the diffraction length $L_d = (2\pi/\lambda)\sigma^2$, where λ is the optical wavelength. The normalized NSE then takes the form

$$i\,\frac{\partial u}{\partial \zeta} + \frac{1}{2}\,\frac{\partial^2 u}{\partial \xi^2} - \frac{d}{2}\,\frac{\partial^2 u}{\partial \tau^2} + \mathrm{sgn}(n_2)N^2|u|^2 u = 0\,. \quad (1)$$

Here the parameter $N^2 = (2\pi\sigma/\lambda)^2 n_0|n_2|I_0$ represents the strength of the Kerr nonlinearity (it will be seen below that N represents the soliton order). The quantity $n_2 I_0$ is the maximum nonlinear index change for an input pulse of peak intensity I_0, and, depending on the sign of n_2, it can be either positive (self-focusing) or negative (self-defocusing). The magnitude of the parameter $d = \mathrm{sgn}(\beta_2)L_d/L_D = (2\pi/\lambda)\sigma^2\beta_2/{T_0}^2$ is the ratio of the diffraction length to the dispersion length ($L_D = {T_0}^2/|\beta_2|$) and represents the relative strengths of dispersion and diffraction. Here β_2 is the group-velocity-dispersion parameter, defined as $\beta_2 = (\partial^2\beta/\partial\omega^2)_{\omega=\omega_0}$, and $\beta(\omega)$ is the propagation constant of the fundamental waveguide mode dispersion. The d parameter can also be either positive or negative, depending on whether the medium is normally ($\beta_2 > 0$) or anomalously ($\beta_2 < 0$) dispersive.

For a linear medium ($N = 0$), the NSE is separable in ξ and τ, leading to solutions whose space- and time-domain features evolve independently of each other. The inclusion of the nonlinearity makes the NSE inseparable in ξ and τ, thereby coupling the behavior in the two domains together. It also makes analytical solutions extremely difficult to achieve; hence our numerical approach. We use the well-known split-step Fourier method[1] to model the evolution of an input Gaussian field of the form

$$u(\xi, \tau, 0) = \exp\left(-\frac{\xi^2}{2} - \frac{\tau^2}{2}\right)\exp[i\phi(\xi, \tau)]\,. \quad (2)$$

Here we use $\phi(\xi, \tau)$ to represent the optical phase modulation, which is typically of the form $\phi(\xi, \tau) = -\xi^2/2f - C\tau^2/2$. Depending on the medium parameters n_2 and β_2, quadratic spatial phase modulation (e.g., a lens of focal length fL_d) may be used, by means of the spatiotemporal coupling provided by the nonlinearity, to control the width of the field in the time domain (the pulse width). Similarly, imposing a frequency chirp ($C \neq 0$) on the input pulse can alter the spatial width of the beam. Because both β_2 and n_2 can be positive or negative, pulse propagation can be classified in four different propagation regimes. These regimes were previously investigated[14] in the context of modulation instability, and it was found that a cw beam can be modulationally unstable in the normal-dispersion regime in both the self-focusing and the self-defocusing cases because of spatiotemporal coupling. It was also shown that the anomalous-dispersion regime of a self-defocusing medium was modulationally stable. We discuss the field behavior in each regime separately with an emphasis on the pulse and beam widths. However, before doing so it is useful to consider spatial and temporal solitons that are exact solutions[15] of Eq. (1) under certain special conditions.

3. SPATIAL AND TEMPORAL SOLITONS

To understand better the effects of a self-focusing nonlinearity ($n_2 > 0$) we should first investigate solutions of the NSE under some simplified conditions. For a relatively wide pulse ($T_0 > 10$ ps) the dispersion length becomes much larger than the diffraction length, such that $d \ll 1$. We can then neglect the third term in Eq. (1) so that Eq. (1) takes the form

$$i\,\frac{\partial u}{\partial \zeta} + \frac{1}{2}\,\frac{\partial^2 u}{\partial \xi^2} + \mathrm{sgn}(n_2)N^2|u|^2 u = 0\,. \quad (3)$$

For a self-focusing medium ($n_2 > 0$) Eq. (3) has exact solutions,[15] known as spatial solitons, for integer values of N such that N represents the order of the spatial soliton. The fundamental soliton ($N = 1$) is characterized by an unchanging, hyperbolic secant profile sech(ξ) that occurs because the nonlinearity and the diffraction add phase curvatures of opposite sign to the field that exactly balance each other. Higher-order solitons ($N > 1$) do not maintain their shapes consistently but return to them periodically with a period $\zeta = \pi/2$. Between these revivals the FWHM of the beam passes through a minimum that gets narrower and closer to $\zeta = 0$ as the order of the soliton increases. After the beam narrows, it splits into a multipeaked structure before returning to its initial shape at $\zeta = \pi/2$. This behavior is demonstrated in Fig. 1(a) for an $N = 3$ spatial soliton.

A similar situation occurs for relatively wide input beams ($\sigma > 1$ cm) with short pulses ($T_0 < 1$ ps) for which the plane-wave approximation holds and the diffraction length becomes much larger than the dispersion length ($d \gg 1$). In this case we renormalize the NSE to the dispersion length. When the diffractive term is dropped, Eq. (1) becomes

$$i\,\frac{\partial u}{\partial \zeta'} - \frac{\mathrm{sgn}(\beta_2)}{2}\,\frac{\partial^2 u}{\partial \tau^2} + \mathrm{sgn}(n_2)N'^2|u|^2 u = 0\,. \quad (4)$$

Equation (4) also models pulse evolution in fibers for which diffractive effects are controlled by the fiber geo-

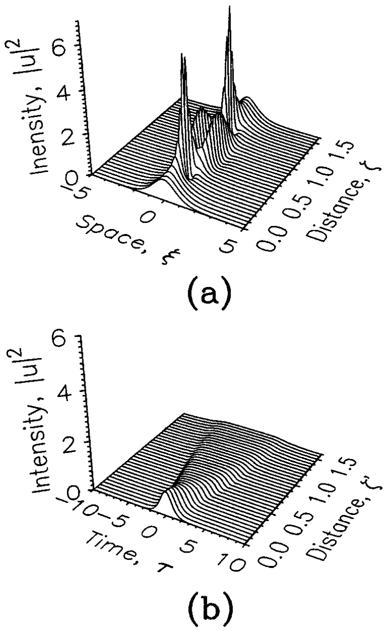

Fig. 1. Results of one-dimensional simulations showing the two types of behavior described by the NSE for a self-focusing nonlinearity. For an input Gaussian field with $N = 3$ we have (a) the evolution through compression, splitting, and recovery of a bright spatial soliton and (b) the monotonic broadening of a pulse in the normal-dispersion regime.

metry. Here the parameters $N'^2 = (2\pi T_0{}^2/\lambda\beta_2)n_0|n_2|I_0$ and $\zeta' = z/L_D$ reflect the new normalization. If n_2 and β_2 have the same signs, soliton solutions of Eq. (4) do not exist, and the pulse monotonically broadens, as can be seen from Fig. 1(b) for $N' = 3$. However, if n_2 and β_2 have opposite signs, Eq. (4) becomes isomorphic to Eq. (3), except that it describes the temporal evolution of the field. The NSE then supports temporal solitons. Higher-order temporal solitons also recover their input shape periodically but go through an initial compression stage, an effect known as soliton-effect pulse compression.[1] The evolution of a third-order temporal soliton is identical to that in Fig. 1(a), except that the ξ direction now represents time and the propagation distance is normalized to the dispersion length. As can be seen there, even a nonlinearity as low as $N = 3$ can generate a significantly compressed pulse. We can also see that the compression is followed by pulse splitting and then a return to its initial shape, as discussed above.

4. SELF-FOCUSING NONLINEARITY

For a narrow beam of ultrashort pulses, $L_d \approx L_D$ and $d \approx 1$. We can then make neither of the approximations discussed above and so must use Eq. (1) to describe the field evolution. One might guess that, inasmuch as anomalous dispersion and diffraction both lead to field compression in the presence of the self-focusing nonlinearity, when both effects are included the degree of compression would increase. In fact it does, and, for nonlinearities such that $N > \sqrt{2}$, self-focusing eventually leads to wave collapse and a breakdown[16] of our model [in reality the wave collapse is averted by higher-order dispersion and self-steepening terms neglected in Eq. (1)]. The dynamics of the wave collapse have been investigated by several authors[16–18] and are beyond the scope of this paper.

To show how spatiotemporal coupling can be used to advantage we apply a quadratic spatial phase modulation (a thin lens) to the input field by choosing $\phi(\xi, \tau) = -\xi^2/2f$ in Eq. (2). In Fig. 2 we plot the spatially averaged pulse width (FWHM) and the time-averaged beam width (FWHM) as a function of propagation distance for a range of focal lengths f with $N = 3$ and $d = -1$. As Fig. 2 shows, a focusing modulation ($f > 0$) can hasten the collapse, whereas a defocusing modulation can either delay or eliminate it entirely, depending on the modulation amplitude, with larger amplitudes (smaller values of f) being required for overcoming larger nonlinearities. The noteworthy point here is that the time-domain behavior is altered with a spatial manipulation. The mechanism at work is fairly simple. The spatial phase modulation for $f < 0$ merely spreads energy out (or helps to concentrate it for $f > 0$) from the center of the field, thereby reducing (or enhancing) the nonlinearity-induced phase curvature of the field, which is the cause of the collapse. This result is analogous to that of Ref. 16, in which the self-focusing of chirped pulses in the anomalous-dispersion regime was numerically and analytically investigated.

In the normal-dispersion regime and for negligible diffraction (plane-wave approximation) the interaction of the dispersion and the nonlinearity leads to a monotonic pulse spreading, as shown in Fig. 1(b). However, the inclusion of the diffractive term can lead to a modest

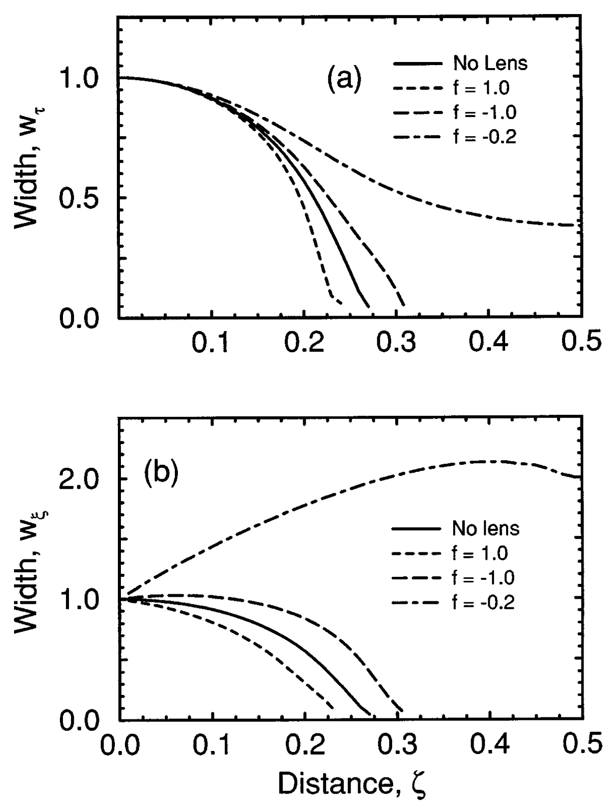

Fig. 2. To show the influence of spatial phase modulation on the wave collapse that occurs for $N = 3$ and $d = -1$, the integrated pulse and beam widths as a function of propagation distance are plotted. In (a) the pulse compression can be increased or decreased depending on the type of lens employed, and in (b) the beam width behaves in a corresponding fashion.

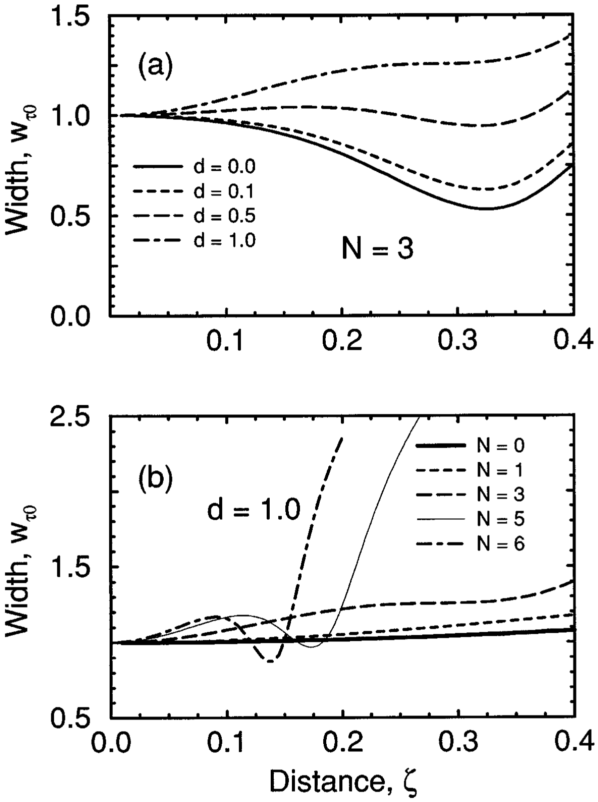

Fig. 3. The amount of localized pulse compression in the normal-dispersion regime depends on both the amount of dispersion and the strength of the nonlinearity. The normalized pulse width at $\xi = 0$ is plotted, showing that (a) the amount of localized compression decreases as the strength of the dispersion is increased and that (b) for $d = 1$, compression of the pulse below its input width requires a strong nonlinearity ($N = 5$).

degree of localized pulse compression. We see this in Fig. 3(a), where we plot the pulse width $w_{\tau 0}$ (FWHM) at the beam center $\xi = 0$, normalized to its input value, as a function of propagation distance for a range of dispersion strengths. We can understand this effect by recalling that higher-order ($N > 1$) spatial solitons in waveguides undergo periodic beam narrowing (and a corresponding increase in intensity) and that the distance to the first minimum decreases as N increases. In the nondispersive case ($d = 0$) an input pulse with $N > 1$ can be viewed as a continuous series of spatial solitons ranging from zeroth order at the wings of the pulse to order N at its center. As the center has the highest order, it will spatially narrow at a shorter distance than the wings. As a result, the pulse appears compressed because the center has become more intense whereas the wings have remained virtually unchanged. When normal dispersion is included for a self-focusing nonlinearity ($n_2 > 0$) the effect is to spread power from the peak of the pulse to the wings. For weak dispersion ($d \approx 0.1$) this effect is minimal compared with the spatial soliton effects; thus the compression is strongest. As the strength of the dispersion is increased, the effect becomes more important, until at $d = 1$ a large nonlinearity ($N = 5$) is required for even a minimal (3%) reduction in pulse width [Fig. 3(b)] at the beam center.

When we view the entire beam by integrating over ξ before measuring the pulse width we find that if $d > 0$ the average pulse width increases and if $d < 0$ the pulse is compressed, provided that $|N^2/d| > 1$, which is precisely the result obtained for the one-dimensional case described by Eq. (4). However, spatiotemporal coupling still plays a role in the field behavior. The spatial self-focusing dominates the field behavior initially, creating the large peak intensity and localized pulse compression of Fig. 3(a). The large peak intensity creates a large nonlinearity-induced field curvature, which is what fuels the pulse broadening. Consequently, the more dominant the initial self-focusing, the broader the pulse will be at large ζ. To understand the influence of spatiotemporal coupling we should plot the pulse width as a function of dispersion length for a constant value of N^2/d. Thus, from Fig. 4(a), where we plot the spatially averaged pulse width w_τ as a function of $\zeta' = \zeta d$ for two dispersion strengths ($d = 0.1$, 1) and $|N^2/d| = 16$, we see that the pulse is broader at $\zeta' = 0.4$ for the $d = 1$ case because the spatial self-focusing is initially ten times stronger than in the $d = 0.1$ case. Similarly, a strong initial dispersion, which reduces the peak-field strength quickly, will in turn reduce the strength of the spatial self-focusing. Referring to Fig. 4(b), where we plot the temporally averaged beam width w_ξ normalized to its input value for $N = 3$ and a range of dispersion strengths, we see that, as the strength of the dispersion is increased, the over-

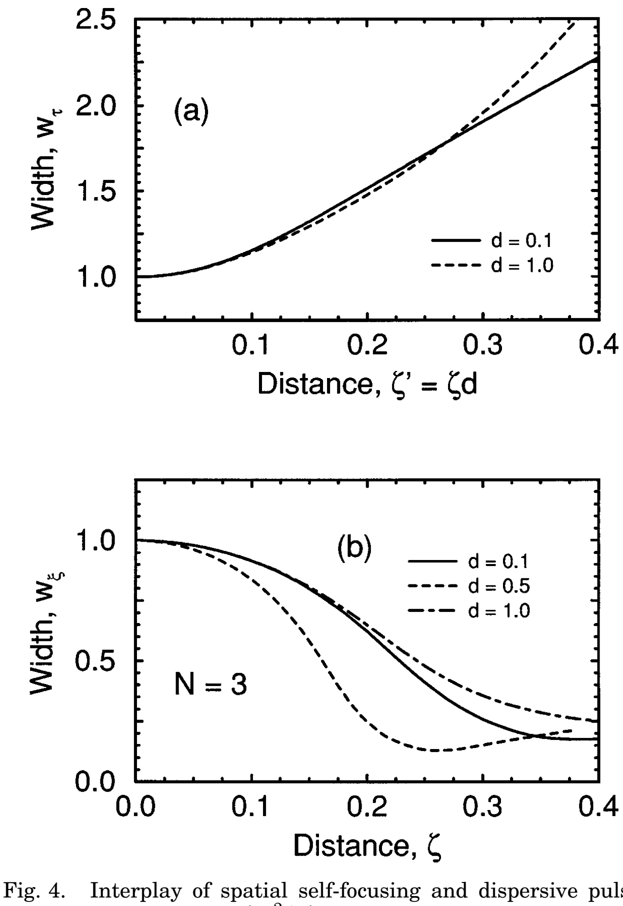

Fig. 4. Interplay of spatial self-focusing and dispersive pulse broadening. In (a), for $|N^2/d| = 16$ the integrated pulse width as a function of dispersion length, the stronger self-focusing (for $d = 1$) eventually creates a broader pulse. In (b) for $N = 3$, increasing the strength of the dispersion first enhances ($d = 0.5$) the self-focusing and then saturates it so that for $d = 1$ the self-focusing is actually weaker.

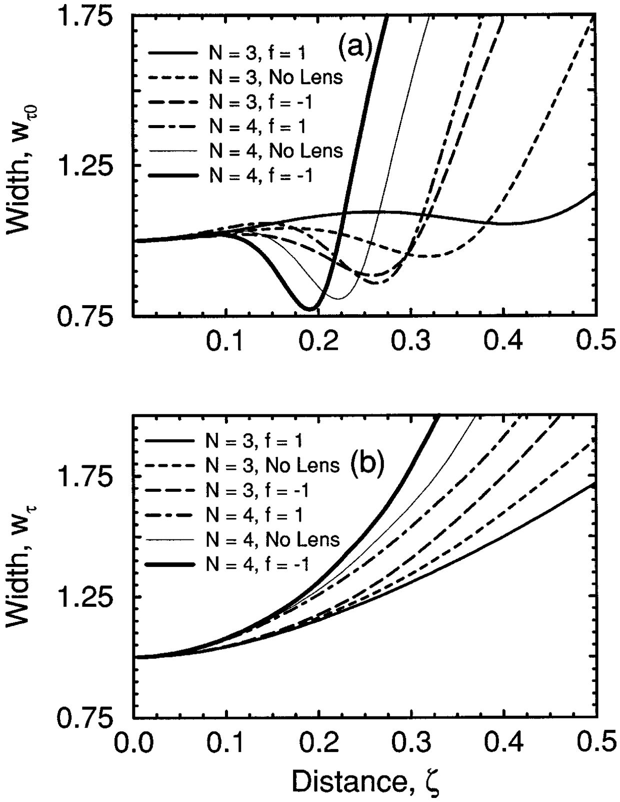

Fig. 5. Influence of spatial phase modulation on the temporal behavior for $d = 0.5$ and two different nonlinearites. In (a) the plot of the pulse width at $\xi = 0$ shows that, depending on the lens, the localized compression can be either enhanced or suppressed with spatial phase modulation, whereas in (b) the localized compression is seen to enhance the integrated-pulse broadening.

all beam-narrowing influence of the self-focusing medium is first reduced before increasing again. The reason for this is that the strength of the nonlinear pulse broadening depends on N^2/d, whereas the distance over which its effects become important is proportional to d; consequently the beam-broadening influence of the dispersion reaches a maximum at approximately $d = 0.5$.

We can further exploit the spatiotemporal coupling with spatial phase modulation and either enhance or suppress the pulse compression. As seen from Fig. 5(a), where we plot $w_{\tau 0}$ as a function of ζ, the degree of localized pulse compression changes with both the modulation amplitude and the strength of the nonlinearity. In Fig. 5(b) we see that the effect of the modulation on the spatially integrated pulse width w_τ is as expected from the discussion in the preceding paragraph. Whatever enhances (reduces) $w_{\tau 0}$ for small ζ will reduce (enhance) w_τ at large ζ'. Also note that, with the spatial phase modulation, localized pulse compression can occur over a wide range of nonlinearities where it would not occur at all in its absence. However, even with a large modulation amplitude and a weak positive dispersion ($d = 0.1$) there is still no spatially averaged pulse compression.

5. SELF-DEFOCUSING NONLINEARITY

In the self-defocusing case with the plane-wave approximation the roles of the two dispersion regimes are reversed. The nonlinearity now works with anomalous dispersion, and the pulse broadens exactly as it does in the normal-dispersion regime of a self-focusing medium [Fig. 1(b)]. Conversely, the normal-dispersion regime with a defocusing nonlinearity supports temporal solitons [Fig. 1(a)]. The picture becomes more interesting when we include diffraction. In the anomalous-dispersion case, diffraction and dispersion work together to broaden the field in both dimensions. As a result, for small ζ the pulse at the beam center broadens more quickly than in the one-dimensional case, but, because this reduces the strength of the nonlinearity, the effect for large ζ is a more slowly broadening spatially averaged pulse, as shown in Fig. 6. Spatial phase modulation in this case can slow the rate of pulse broadening, but at the expense of broadening the beam spatially.

In the absence of diffraction the natural effect of normal dispersion interacting with the defocusing nonlinearity is to compress the pulse, the compression factor becoming larger as the nonlinearity is increased. Similarly, cw simulations show that the effect of diffraction interacting with the nonlinearity is to broaden the beam. It is therefore not surprising that, when the diffractive effects of a waveguide are included with a defocusing nonlinearity, the result is a more moderate pulse compression (rather than a wave collapse), as shown in Fig. 7(a). The reason for this is that, although the dispersion and the nonlinearity are acting to bring energy in from the temporal wings of the field, the diffraction is using the nonlinearity to move energy out to the spatial wings, thereby

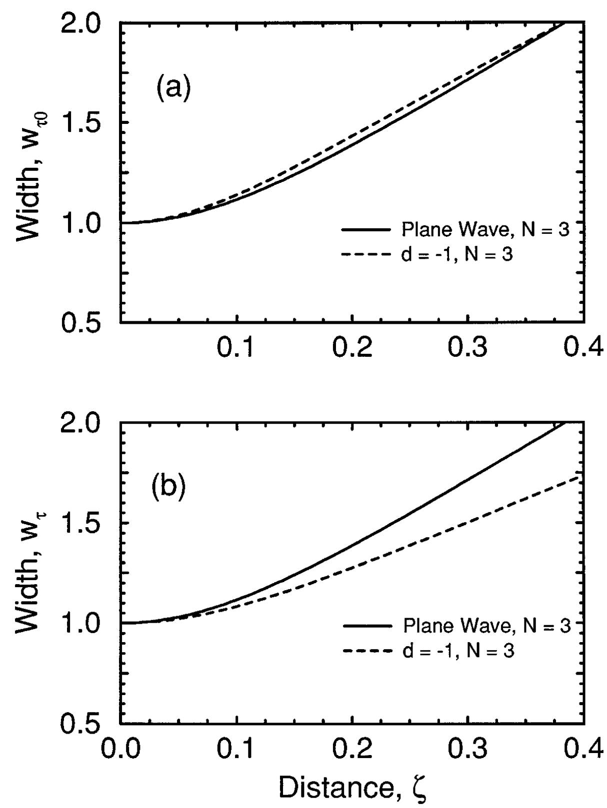

Fig. 6. In the anomalous-dispersion regime with a self-defocusing nonlinearity the pulse and the beam broaden monotonically. In (a) including diffraction in the model is seen initially to slightly increase the pulse width at the beam center, whereas in (b) the effect of diffraction on the spatially integrated pulse width is the opposite: the rate of broadening is decreased.

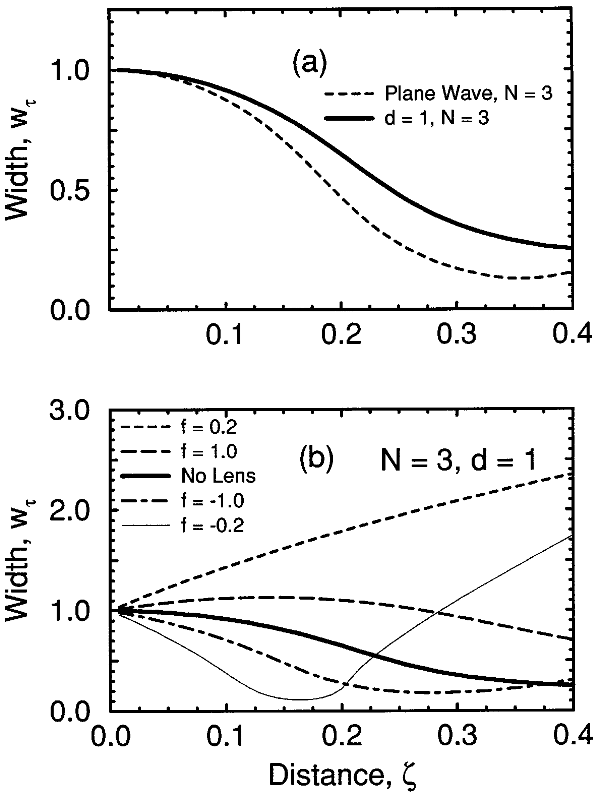

Fig. 7. In the normal-dispersion regime with a self-defocusing nonlinearity the pulse compresses and the beam broadens. In (a) the effect of including diffractive effects on the spatially integrated pulse is to reduce the broadening, and in (b) spatial phase modulation is employed to enhance the pulse compression.

reducing the peak field strength and thus the pulse compression. We can counteract the self-defocusing-induced beam spreading by imposing a focusing spatial phase curvature on the input field that the diffraction must overcome before it can reduce the peak power. Thus, because of the spatiotemporal coupling, we are again able to enhance (or reduce) the pulse compression with spatial phase modulation. As shown in Fig. 7(b), with a very tight focus such as $f = 0.5$ we can achieve even greater pulse compression than that obtained without diffraction.

Another interesting aspect of the spatiotemporal coupling that occurs in planar waveguides is the counterintuitive result that, as the strength of the defocusing nonlinearity is increased, the spatial width of the field at the peak of the pulse can be reduced (the beam can be focused), as evidenced by the data in Fig. 8. In Fig. 8(a) we plot the beam width (FWHM) at the pulse peak $w_{\xi 0}$ normalized to its input value as a function of the propagation distance for $d = 2$ and a range of beam intensities. We see that, except for the $N = 2$ case, there is a secondary minimum in $w_{\xi 0}$ that gets deeper as the strength of the nonlinearity is increased. Were we to plot the analogous parameter for the temporal FWHM at the beam center, we would find that the (spatial) secondary minima occur at approximately the same position as the point of maximum temporal compression. Of course for $d = 2$ and $N = 2$ there is no temporal compression, which explains why there is no secondary $w_{\xi 0}$ minimum either. The dependence of the spatial width on the pulse compression takes an interesting turn when we consider the effect of increasing dispersion, as in Fig. 8(b). Here we again plot the beam width at the pulse peak $w_{\xi 0}$ as a function of propagation distance but for a constant nonlinearity ($N = 4$) and for dispersion parameters ranging from 0 to 10. We again see the secondary minima associated with the point of maximum pulse compression. Because the distance to the point of maximum temporal compression is roughly proportional to d, and this is also the distance over which self-defocusing must act to spread the field, one might expect the minimum $w_{\xi 0}$ to decrease as the strength of the dispersion increases. But, inasmuch as the maximum pulse compression increases as N^2/d, the temporal compression weakens as the strength of the dispersion is increased. Therefore, as we see from Fig. 8(b), the minimum $w_{\xi 0}$ actually occurs at $\sim d = 5$, where the pulse is compressed quickly and strongly enough to dominate the self-defocusing. This is also the dispersion for which the beam is broadest at large ζ.

By the same reasoning, a temporal phase modulation (frequency chirp) that will enhance (suppress) the pulse compression will also enhance (suppress) the localized beam narrowing at small ζ. This behavior is shown in Fig. 9(a), where we plot the normalized beam width at the pulse peak $w_{\xi 0}$ for $N = 3$ and a range of chirp parameters, C is -5 to 5. As before, when we observe the effect of modulation on the temporally integrated beam width at large ζ we see that the effect of the spatiotemporal coupling on the field behavior is the opposite of that ob-

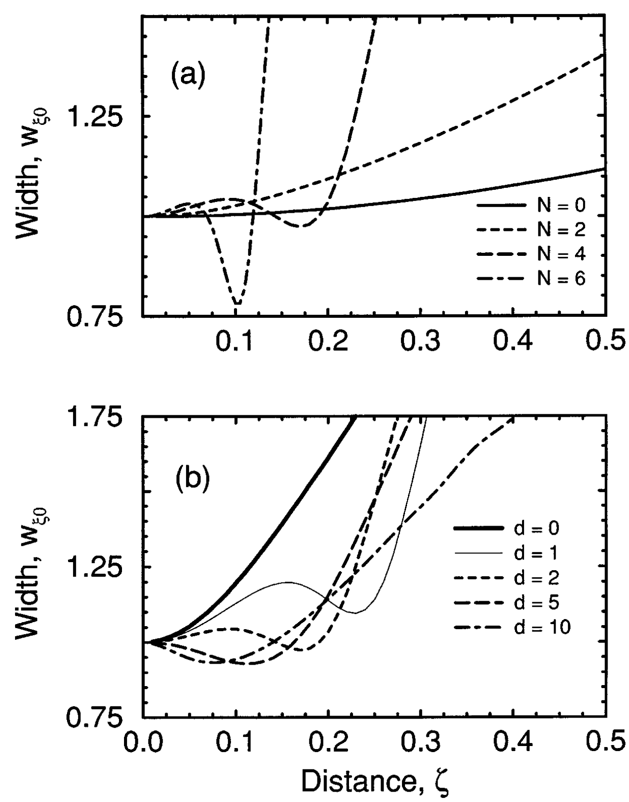

Fig. 8. To show the effect of the N and d parameters, the width (FWHM) of the spatial intensity distribution through the center of the pulse ($\tau = 0$) normalized to its input value is plotted as a function of propagation distance. In (a) the effect of increasing the strength of the nonlinearity is to increase the localized spatial narrowing, whereas in (b) the dispersion-induced enhancement of the localized narrowing saturates near $d = 5$.

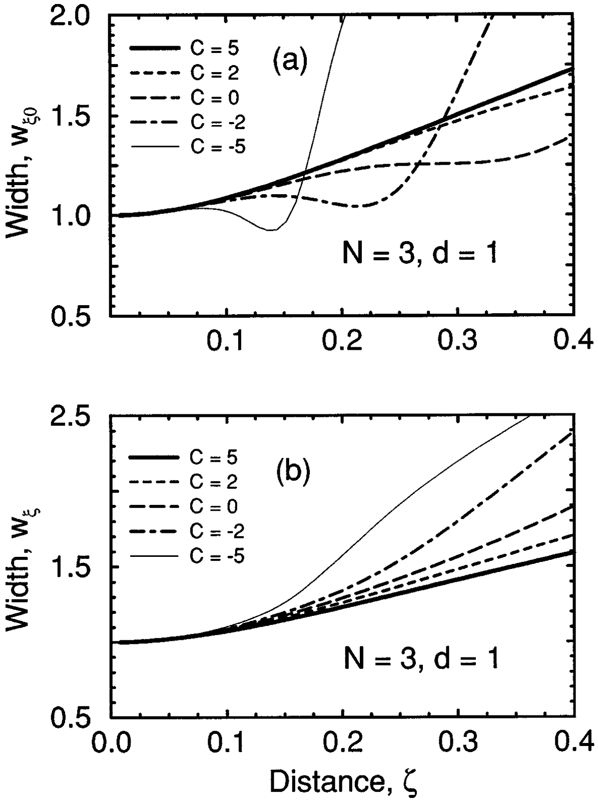

Fig. 9. Temporal phase modulation can be used to control the beam width in a self-defocusing medium. In (a) an upchirp ($C > 0$) is seen to reduce the localized beam narrowing that occurs at $\tau = 0$, whereas a downchirp ($C < 0$) is seen to enhance it. In (b) the effect of chirp on the time-integrated beam width is such that an upchirp leads to a slower beam broadening and a downchirp increases the beam width. In an anomalously dispersive medium the effects of the upchirp and the downchirp would be reversed.

served at the pulse peak for small ζ. In Fig. 9(b) we plot the time-integrated beam width w_ξ for the same parameters as in Fig. 9(a) and see that at large ζ the effect of positive chirp is to reduce the influence of self-defocusing, whereas the effect of negative chirp is to enhance it. In neither case, however, does the modulation come close to eliminating the influence of the nonlinearity on the beam width.

6. CONCLUSIONS

We have presented an overview of the effects of spatiotemporal coupling on the behavior of ultrashort pulses traveling in nonlinear planar optical waveguides. We have seen that in a waveguide with $n_2 > 0$ and $\beta_2 > 0$ a strong nonlinearity can bring energy in from the spatial wings of the field faster than the normal dispersion can move it to the temporal wings, thus making a localized pulse compression possible in the normal dispersion regime. This initial compression at the center of the beam, however, creates an enhanced pulse broadening at larger propagation distances. Further, as the initial compression is driven by spatial self-focusing, a focusing spatial phase modulation will enhance both the initial compression and the eventual pulse broadening that occurs at large ζ. Conversely, a defocusing spatial phase modulation will reduce both the initial compression and the later broadening. We have also seen that with $n_2 < 0$ and $\beta_2 > 0$ a strong nonlinearity can bring energy in from the temporal wings faster than the diffraction can spread it to the spatial wings, making a localized beam narrowing in a self-defocusing medium possible. This initial narrowing at the peak of the pulse fuels an even greater self-defocusing at large distances, which may be enhanced or reduced with temporal phase modulation (frequency chirp). In both cases the mechanism is the same: a strong compressive effect initially overpowers a broadening effect, which in turn strengthens the eventual broadening that occurs at larger propagation distances.

There is a variety of materials and devices in which nonlinearity-induced spatiotemporal coupling plays an important role. This coupling will also be important in materials that exhibit Kerr-like nonlinearities, such as the resonant nonlinearities of semiconductors and semiconductor-doped glasses. As discussed above, spatiotemporal coupling is behind the self-mode locking[6–8] of the new generation of ultrafast solid-state lasers ($Ti:Al_2O_3$, $Cr:LiSrAlF_6$, etc.), and further developments in this area necessarily require a clear understanding of this mechanism. A proposed ultrafast pulse-shaping technique also exploits the spatiotemporal coupling provided by the Kerr nonlinearity.[19] Another application in which spatiotemporal coupling should be important is in Z-scan measurements of the nonlinearity of dispersive materials by ultrashort pulses, particularly for thicker samples. Both numerical and experimental investigations in this area are now under way.

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