Steering of optical beams in nonlinear Kerr media by spatial phase modulation

Andrew T. Ryan and Govind P. Agrawal

The Institute of Optics, University of Rochester, Rochester, New York 14627

A simple scheme to steer optical beams is proposed. The basic idea is to impose a sinusoidal phase modulation on the optical beam and then propagate it in a nonlinear Kerr medium. Spatial phase modulation splits the input beam into multiple subbeams, while the nonlinear medium shapes a particular subbeam into a spatial soliton in such a way that most of the beam power appears in a narrow beam whose direction can be controlled by changing the modulation parameters. We present numerical results showing how spatial phase modulation can be used to alter the path of an optical beam propagating in a nonlinear Kerr medium.

In both the temporal and spatial domains the multidimensional nonlinear Schrödinger equation (NLSE) has long been a useful tool for describing the behavior of optical fields in nonlinear dispersive media.[1,2] It has proved valuable in the description of such diverse phenomena as pulse compression, dark soliton formation, and self-focusing of ultrashort pulses. In recent years it has been useful in describing some of the new innovations in beam steering.[3-6] One technique uses area modulation of a second beam to induce a temporal prism in the nonlinear medium, which then deflects the beam.[3] Another two-beam technique uses cross-phase modulation from a pump beam to alter the phase profile of a probe beam and so induce a deflection.[4] Others have employed single beams with asymmetric power profiles, which resulted in self-bending on propagation.[5] Another technique uses the properties of dark solitons for beam steering.[6] There are also a variety of techniques for steering beams in linear media.[7-11] The technique that we propose here employs spatial phase modulation of a beam entering a nonlinear medium and shows that high-efficiency beam steering is possible. Spatial phase modulation splits the input beam into many subbeams, while the nonlinear medium shapes a particular subbeam into a spatial soliton in such a way that most of the beam power appears in a narrow beam whose direction can be controlled by changes in the modulation parameters.

We model beam propagation with the NLSE in the dispersionless (cw or quasi-cw) approximation by using the well-known split-step Fourier method.[1] Spatial transverse coordinates ξ and η are normalized to the input beam width σ, and the propagation distance ζ is measured in units of the diffraction length, $L_d = (2\pi/\lambda)\sigma^2$, where λ is the optical wavelength. The normalized NLSE then takes the form

$$i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right) + N^2|u|^2 u = 0, \qquad (1)$$

where the parameter $N = (2\pi\sigma/\lambda)\sqrt{n_0 n_2 I_0}$ represents the strength of Kerr nonlinearity. The quantity $n_2 I_0$ represents the maximum nonlinear index change for

an input beam of peak intensity I_0 in a medium of linear refractive index n_0 and nonlinear index parameter n_2. We have performed simulations for both one and two transverse dimensions.

We focus first on the case of one transverse dimension so that the results are applicable mainly to planar waveguides. The NLSE is solved for a phase-modulated input beam having a spatial profile

$$u(\xi, 0) = \exp(-\xi^2/2)\exp[i\phi(\xi)]. \qquad (2)$$

For the case of sinusoidal phase modulation, $\phi(\xi)$ has the form

$$\phi(\xi) = \phi_0 \sin(2\pi p\xi + \delta), \qquad (3)$$

where ϕ_0 is the amplitude of the modulation, p is the spatial modulation frequency, and δ is a constant phase shift. By using a standard Bessel-function expansion of the modulation term, one can see that phase modulation breaks up the input beam into multiple subbeams propagating at different angles (Raman–Nath scattering[7]):

$$u(\xi, 0) = \exp(-\xi^2/2)\sum_{-\infty}^{\infty} J_m(\phi_0)\exp[im(2\pi p\xi + \delta)]. \qquad (4)$$

In this form it is clear that the amplitude of each subbeam is determined through the modulation amplitude ϕ_0 and that the steering angles are determined from the modulation frequency p. Thus our numerical simulations represent the interaction and evolution of these initially intersecting subbeams as they propagate through a Kerr medium. We show that, with the proper choice of modulation parameters p, ϕ_0, and δ, we can steer most of the initial beam power into just one of the subbeams.

The numerical simulations for various values of δ show that beam steering is quite sensitive to the phase shifts δ. The beam profile remains symmetric only for $\delta = \pi/2$, as expected from Eq. (3). For other values of δ the beam steers to one side in an asymmetric manner. Specifically, as δ is increased from $\pi/2$ to π, the beam is deflected to the left
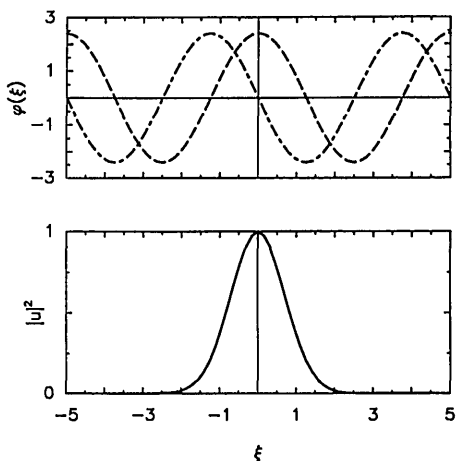
Fig. 1. Spatially modulated phase profiles for $\delta = \pi/2$ (dashed curve) and $\delta = \pi$ (dotted–dashed curve) with $p = 0.2$. The bottom plot shows for comparison the Gaussian intensity profile of the input Gaussian beam.

(it is deflected to the right if δ is decreased to 0). This behavior can be understood by reference to Fig. 1, which shows the modulation function $\phi(\xi)$ and the intensity profile of the beam on the same horizontal scale for $p = 0.2$. When $\delta = 0$ or $\delta = \pi$, phase modulation is nearly linear and is equivalent to placing a prism in the beam path, resulting in beam deflection. By contrast, the phase shift $\delta = \pm\pi/2$ is nearly equivalent to placing a lens in the beam path, as it produces a nearly quadratic variation of the phase front. The amount of power deflected into a given subbeam also depends on δ and is optimal for $\delta = 0$ or $\delta = \pi$, depending on whether the beam is steered toward the right or the left.

As seen from Eq. (4) the amount of power initially in a subbeam also depends critically on the modulation amplitude ϕ_0 through the factor $J_m(\phi_0)$. Indeed, for $\phi_0 \lesssim 1$ there are only three ($m = 0, \pm1$) beams with any significant power. But the ideal choice of modulation depth is $\phi_0 = 2.405$, the first zero of J_0. For this choice of modulation depth, phase modulation leaves no power in the central undeflected portion of the beam. For higher modulation amplitudes not only will some power creep back into the central subbeam but more power will leak into the higher-order subbeams at the wider angles. For low modulation amplitudes, the modulation has the effect of shedding some of the power from the central beam. The impact of the modulation is also highly dependent on the modulation frequency. As is clear from Eq. (4), the modulation frequency determines the transverse component k_x of the subbeam wave vector (e.g., the mth subbeam has a transverse wave vector of $2m\pi p/\sigma$). The initial steering angle of the mth subbeam is thus given by $\theta_m = \tan^{-1}(k_x/k)$. In terms of the beam parameters the steering angle becomes $\theta_m = \tan^{-1}(mp\lambda/\sigma)$. For very low modulation frequencies ($p \ll 1$), subbeams diverge so slowly that the effect of the modulation becomes irrelevant. By contrast, for high modulation frequencies ($p > 1$), subbeams may diverge so rapidly that the paraxial approximation used to derive the NLSE becomes suspect. Even for $p < 1$ the steering

angle θ_m for higher orders becomes large enough to violate the paraxial approximation (expected to hold for $\theta_m < 20°$), particularly for $\sigma \sim \lambda$. In general, for beam widths considerably larger than λ, the paraxial approximation remains valid as long as only lower-order beams carry significant power. We choose modulation frequencies in the range $0.1 < p < 1$ such that the paraxial approximation remains valid.

All the above discussion is quite general since it refers to the initial effect of phase modulation; it can be applied to propagation in both linear and nonlinear media. What we now discuss is the effect of the nonlinear medium on the phase-modulated optical field in a one-dimensional (waveguide) geometry. Figure 2 compares the beam evolution in a nonlinear medium for three values of N (or peak intensity I_0). As seen in Fig. 2(a), for low powers the steered beam diverges on propagation since the medium is nearly linear. But as the power is increased, the primary subbeam is capable of solitonlike propagation, whereas the remaining lower-power subbeams simply diverge. Thus the majority of the beam power is steered in a different direction. We can return again to Eq. (4) to estimate the value of N for which
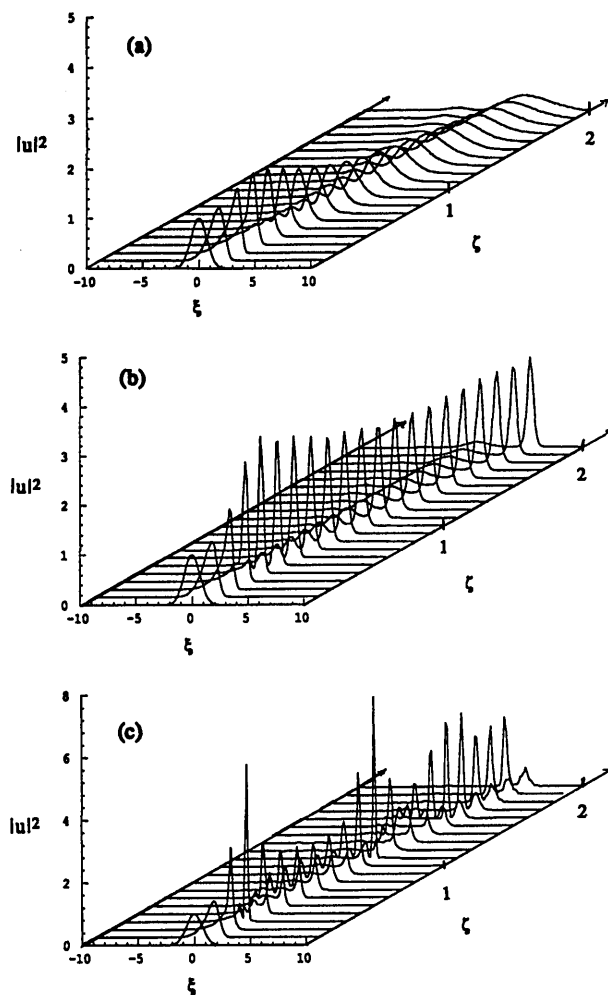

Fig. 2. Evolution of a phase-modulated Gaussian beam in a Kerr medium for three different peak intensities: (a) $N = 0$, (b) $N = 2$, and (c) $N = 3$. The modulation parameters are $p = 0.2$, $\phi_0 = 2.405$, and $\delta = 0$.

the mth-order subbeam will propagate as a soliton. Since the peak amplitude of the subbeam is given by $J_m(\phi_0)$ and the soliton order N_m of the subbeam is linear in field amplitude, we require $N_m = NJ_m(\phi_0) > 1/2$ for soliton propagation of the mth subbeam.[1] For $\phi_0 = 2.405$, $J_1(\phi_0) \approx 0.52$, and $N_1 > 1/2$ when $N > 0.96$. So for $N \gtrsim 0.96$ the first-order subbeam should propagate as a soliton, although for such a low N_1 it would be quite broad. Ideally one would like a narrow subbeam for which a higher N_1 is required. Fundamental solitons will still be obtained for $N_1 < 3/2$, and the higher the N_1 the narrower the subbeam. For $\phi_0 = 2.405$, $N_1 < 3/2$ when $N < 2.88$. Thus the first-order subbeam can form a soliton for values of N in the range $0.96 < N < 2.88$.

One should also ensure that the second-order subbeam does not form a soliton if the objective is to obtain most of the power in a single steered beam. It would appear that this requirement severely limits the range of N since N_2 can exceed $1/2$ when $N > 1.16$. However, such a simplistic argument assumes that the subbeams propagate independently. In reality, first- and second-order subbeams interact strongly during propagation until they are separated from each other. Our numerical simulations automatically include mutual interaction of the subbeams with proper phase. As seen in Fig. 2(b), the second-order subbeam does not form a soliton even for values of N as large as 2. For larger values of N [$N = 3$ in Fig. 2(c)], the second-order subbeam does form a soliton. However, as seen in Fig. 2(c), the two solitons corresponding to $m = 1$ and $m = 2$ pass through each other and keep propagating as solitons after their collision. Our numerical simulations indicate that the optimal beam power for steering the first-order subbeam is such that the parameter N is close to 2.

We now consider beam steering in bulk nonlinear media by including both transverse dimensions in our numerical simulations. The two-dimensional analog of Eqs. (2) and (3) is

$$u(\xi, \eta, 0) = \exp[-(\xi^2 + \eta^2)/2]\exp[i\phi_1 \sin(2\pi p_1\xi + \delta_1) + i\phi_2 \sin(2\pi p_2\eta + \delta_2)], \quad (5)$$

where p_1 and p_2 are spatial phase-modulation frequencies in the two directions. One can steer the beam along one axis by setting either ϕ_1 or ϕ_2 equal to zero. In the general case the beam can be steered in any direction by choice of ϕ_1, ϕ_2, δ_1, δ_2, p_1, and p_2 appropriately. Most of the above discussion can be easily carried over to the two-dimensional case. The main difference comes from the fact that now the first-order subbeam can collapse catastrophically if the subbeam power exceeds the critical power and the medium length exceeds the so-called self-focusing distance. However, one can easily avoid the collapse problem in practice by choosing the medium length shorter than the self-focusing distance since the objective is to counteract diffraction broadening. In essence, the nonlinearity is used only to control the beam size.

These results have an interesting temporal analog. The NLSE for cw beam propagation in a one-dimensional (waveguide) geometry is mathematically equivalent to the NLSE for short optical pulses propagating in single-mode fibers.[1] The interpretation of our results in the temporal domain is as follows: a sinusoidal temporal phase modulation of an ultrashort pulse can create subpulses that are either delayed or advanced with respect to the input pulse. These subpulses can of course become solitons with the proper choice of modulation parameters. The width of the input pulse imposes a limit to the amount of delay or advance of the subpulse that is physically possible in the same way that the beam width limits the amount of steering that is actually possible. The limitations on the temporal delay or advance have been observed experimentally by da Silva et al. in a different context.[12]

In conclusion, we have proposed a simple scheme to steer optical beams. The basic idea is to impose a sinusoidal phase modulation on the optical beam and then propagate it in a nonlinear Kerr medium. Spatial phase modulation splits the input beam into many subbeams, while the nonlinear medium shapes a particular subbeam into a spatial soliton in such a way that most of the beam power appears in a narrow beam whose direction can be controlled by changes in the modulation parameters. To our knowledge, this kind of beam steering has not yet been observed experimentally. The basic concept can be demonstrated with the use of the Kerr nonlinearity of CS_2. Practical applications would require the use of a solid-state medium.

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