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# Generalized Bloch–Maxwell formulation for semiconductor lasers

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A perturbative method, is used, which includes, for the first time, local field effects, to obtain a set of generalized Bloch equations based upon averaging of the Bloch equations over the intraband distributions by the introduction of new dynamical variables. It is shown that the new set of Bloch equations, taken in the rate-equation limit, and neglecting local field effects, leads to the standard rate equation for the carrier density. Also is shown that the index saturates because of intraband relaxation effects, whereas the gain saturates mainly due to interband transitions. In our approach, the origin of phase-amplitude coupling, manifested through the linewidth enhancement factor, lies in the asymmetric nature of the carrier distribution within the conduction and valence bands.

## 1. Introduction

Considerable interest has been expressed over the last decade in the formulation of the dynamics of semiconductor lasers in terms of Bloch-like equations coupled to the electromagnetic field [1]. This has been due, in part, to the surge of importance of semiconductor lasers in pure and applied science as well as in a plethora of device applications in optoelectronics, and optical communications and optical data processing [2]. It is also due to the relative ease of manipulation that a Bloch–Maxwell-like formulation would provide in predicting and analyzing new physical phenomena and characteristics, as has been the case in nonlinear and quantum optics of gases and vapors [3,4]. Attempts to arrive at an analogous formulation of reduced equations of motion, ab initio, have not been particularly successful [1].

On the other hand, the somewhat phenomenological approach based on the linear variation of the gain and the refractive index with respect to the carrier density, has produced a variety of important and interesting results [5]. Attempts have been made to improve the phenomenological description by using the density-matrix formulation in which the semiconductor laser is modeled as a collection of two-level atoms with a range of transition frequencies, similar to an inhomogeneously broadened two-level system [6–10]. Such an approach has been successful in explaining a number of features. For example, it has led to fundamental understanding and interpretation of gain suppression (so-called nonlinear gain) in semiconductor lasers [6,9], the description of gain and mixing susceptibilities in amplifiers [7,8], and analysis of spectral hole-burning and gain saturation [9,10] in semiconductor lasers. A formulation, similar to that of the optical Bloch equations [3], however, has yet to be presented and justified. A set of such equations would certainly be exceedingly useful in their regimes of validity, in predicting and analyzing new physical effects and possibly unexplored phenomena in semiconductors such as self-induced transparency, superfluorescence, and four-wave mixing.

It is our purpose, here, to introduce a hierarchy of Bloch-like equations, suitable for investigating nonlinear dynamical behavior in semiconductor lasers. To proceed unencumbered, we confine our attention to single-mode operation. In the next section, we introduce the density-matrix formulation of the equations of motion which account for the band structure, and intraband and interband relaxation processes. We use the perturbative method of Graham and Cho [11], to reduce the set of equations for an inhomogeneously broadened, single-mode laser, to a set of Bloch-like equations which do not require treatment of the band-structure details. In sect. 3, it is shown that in the rate-equation limit, the new generalized Bloch equations yield the standard rate equation for the carrier density. The final section is used for discussion, summary, and conclusion.

## 2. Equations of motion

The density-matrix formulation and semiclassical theory are used to express the equations of motion for an inhomogeneous distribution of two-level systems interacting with the electromagnetic field in the electric dipole and rotating-wave approximations [6–10],

$$\frac{\mathrm{d}\rho_{11}}{\mathrm{d}t} = \frac{\mu}{\mathrm{i}\hbar} E_{\mathrm{L}}\rho_{12} - \frac{\mu}{\mathrm{i}\hbar} E_{\mathrm{L}}^{*}\rho_{21} , \tag{1}$$

$$\frac{\mathrm{d}\rho_{22}}{\mathrm{d}t} = -\frac{\mu}{\mathrm{i}\hbar} E_{\mathrm{L}}\rho_{12} + \frac{\mu}{\mathrm{i}\hbar} E_{\mathrm{L}}^{*}\rho_{21} , \tag{2}$$

$$\frac{\mathrm{d}\rho_{12}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(\epsilon_1 - \epsilon_2)\rho_{12} + \frac{\mu}{2\mathrm{i}\hbar} E_{\mathrm{L}}(\rho_{11} - \rho_{22}) . \tag{3}$$

Here, $\rho_{11}$ and $\rho_{22}$ represent the occupation probabilities for electrons in the conduction and valence bands, respectively, for a certain wave vector, $k$, and $\rho_{12}$ is related to the polarization induced by an electron transition between the valence and conduction bands initiated by the local electric field $E_{\mathrm{L}}$, and $\mu$ is the associated matrix element of the transition dipole moment [5]. The energy-level separation for corresponding *k-states* in the conduction and valence bands is given by $\epsilon_1 - \epsilon_2 = \hbar\omega$ in eq. (3), where

$$\epsilon_1(k) = E_{\mathrm{c}}(k) = \hbar^2 k^2/2m_{\mathrm{c}} + E_{\mathrm{g}} , \tag{4a}$$

$$\epsilon_2(k) = E_{\mathrm{v}}(k) = -\hbar^2 k^2/2m_{\mathrm{v}} . \tag{4b}$$

$E_{\mathrm{g}}$ is the gap energy and $m_{\mathrm{c}}$ and $m_{\mathrm{v}}$ are, respectively, the electron effective mass in the conduction and valence bands.

In the plane-wave approximation, the electric field amplitude, $E$, and induced polarization in the medium, $P$, are expressed in terms of slowly-varying amplitudes, $\mathscr{E}$ and $\mathscr{P}$, and the field carrier frequency, $\omega_{\mathrm{L}}$,

$$E = \tfrac{1}{2}\mathscr{E}\exp(-\mathrm{i}\omega_{\mathrm{L}}t) + \text{c.c.} , \tag{5a}$$

$$P = \tfrac{1}{2}\mathscr{P}\exp(-\mathrm{i}\omega_{\mathrm{L}}t) + \text{c.c.} , \tag{5b}$$

where

$$P = \sum \mu(\rho_{12} + \rho_{21}) , \tag{6}$$

and the summation is over the distribution of joint density of states. Equations (1)–(3) can be written in terms of slowly-varying variables using eq. (5) and

$$\rho_{12} = \rho\exp(-\mathrm{i}\omega_{\mathrm{L}}t) . \tag{7}$$

Then,

$$\mathscr{P} = 2\mu\sum\rho . \tag{8}$$

If, for convenience, $\mathscr{P} \equiv 2\mu N_{\mathrm{t}} p$, where $N_{\mathrm{t}}$ is the density of electrons in the absence of injection, then,

$$p = \frac{1}{N_{\mathrm{t}}}\sum\rho . \tag{9}$$

It is convenient to replace the sums in eqs. (6), (8), and (9) by an integral. For any variable, $x$,

$$\langle x\rangle \equiv \frac{1}{N_{\mathrm{t}}}\sum x = \frac{1}{\gamma_{\mathrm{T}}}\int x(\omega)\, D(\omega)\,\mathrm{d}\omega , \tag{10}$$

where $D(\omega)$ is the joint density of states. By introducing the normalized detuning, $\delta = (\omega - \omega_0)/\gamma_{\mathrm{T}}$, where the choice of $\omega_0$ is arbitrary, eq. (10) can be written as

$$\langle x\rangle = \int x(\delta)\, D(\delta)\,\mathrm{d}\delta . \tag{11}$$

Here, $\gamma_{\mathrm{T}}$ is the dipole dephasing rate between the electronic excited state, in the conduction band, and ground state, in the valence band [6], assumed constant over the distribution of states.

The Maxwell wave equation in the plane-wave and the slowly-varying amplitude and phase approximation can now be written as

$$\frac{\partial \mathscr{E}}{\partial z}+\frac{1}{v_g}\frac{\partial \mathscr{E}}{\partial t}=\mathrm{i}\frac{\omega_L \mu N_t}{\epsilon_0 cn}p\,. \tag{12}$$

Here, $n$ is the linear refractive index of the material, $v_g$ is the group velocity and $c$ is the speed of light in vacuum. It is to be observed at this point that the field amplitude, $\mathscr{E}$, which appears in eq. (12) is not identical to the field amplitude, $\mathscr{E}_L$, appearing in eqs. (1)–(3).

We focus attention on the field amplitude, $\epsilon_L$, written here as the slowly-varying amplitude component of $E_L$ which stems from the dipole–field interaction, and is the microscopic field which drives an interband electronic transition, and consists of all fields acting on the particle. It does not contain the oscillating electron's self-field, whereas the macroscopic field $\mathscr{E}$, eq. (12), consists of all fields present [12]. Thus, in order to establish a self-consistent relationship between eqs. (1)–(3) and eq. (12), a transformation is needed to relate $\mathscr{E}_L$ to $\mathscr{E}$, i.e., we require a relation, $\mathscr{E}_L=\mathscr{E}_L(\mathscr{E})$. The correction is small and generally insignificant provided there are very few dipoles, on the average, within a cubic resonance wavelength. However, if the density of dipoles is such that there are many dipoles, on the average, within a cubic wavelength, one must take into account that the dipoles interact with one another via their mutual dipole–dipole interactions, and the effect can be significant [13–15]. With the assumption that there are many dipoles within a cubic resonance wavelength, it has been shown that [12–15]

$$\mathscr{E}_L=\mathscr{E}+(\sigma/\epsilon_0)\mathscr{P}\,, \tag{13}$$

or, from eqs. (8) and (9),

$$\mathscr{E}_L=\mathscr{E}+2\sigma\mu N_t p/\epsilon_0\,. \tag{14}$$

Here, $0\leqslant\sigma\leqslant\frac{1}{3}$, is a structure factor where $\sigma=\frac{1}{3}$ for cubic or spherical microscopic local symmetry at a given dipole, and is less than that value for lower symmetries [12–15,17]. The relation, eq. (13), is often called the "local field correction" [16,17], and is responsible for such phenomena in condensed matter physics, and dense materials, as the Clausius–Mossotti relations [18], and linear and nonlinear spectral shifts [19].

If eq. (14) is used in eqs. (1)–(3) to eliminate $\mathscr{E}_L$, using eq. (7), and upon introducing phenomenological relaxation, dephasing, and pumping terms [10], eqs. (1)–(3) become

$$\mathrm{d}w/\mathrm{d}t=-\gamma_c(w-\bar{w})-\gamma_L(w-w_{th})$$
$$-(\mathrm{i}\mu/\hbar)(\mathscr{E}^*\rho-\rho^*\mathscr{E})$$
$$-2\mathrm{i}\epsilon(\langle\rho\rangle^*\rho-\rho^*\langle\rho\rangle)+\Lambda\,, \tag{15}$$

$$\mathrm{d}\rho/\mathrm{d}t=-\gamma_T(1+\mathrm{i}\Delta)\rho-\mathrm{i}\gamma_T\delta\rho$$
$$+(\mu/2\mathrm{i}\hbar)\mathscr{E}w-\mathrm{i}\epsilon\langle\rho\rangle w\,, \tag{16}$$

where $\Delta=(\omega_0-\omega_L)/\gamma_T$ is the detuning parameter,

$$\epsilon=\frac{2\sigma}{\epsilon_0}\frac{\mu^2}{\hbar}N_t\,, \tag{17}$$

$$w=\rho_{11}-\rho_{22}\,, \tag{18}$$

$w_{th}$ is the value of $w$ in thermal equilibrium and $\bar{w}$ is the effective steady-state value. The strength of the near dipole–dipole interaction is given by $\epsilon$, eq. (17), and appears in eqs. (15) and (16) explicitly due to the introduction of eq. (14) into eqs. (1)–(3). In introducing the non-hamiltonian contributions to eqs. (15) and (16), $\gamma_c$ represents interband relaxation, assumed equal for the entire distribution, and $\gamma_T$ is the dipole dephasing rate for electronic transitions, also assumed constant over the distribution of states. $\gamma_L$ represents the population relaxation rate and is related to the interband transitions occurring spontaneously through phenomena such as spontaneous emission. Typically, $\gamma_L\sim 10^9\ \mathrm{s}^{-1}$, but $\gamma_c$ and $\gamma_T$ are $10^{13}\ \mathrm{s}^{-1}$ in semiconductor lasers. The last term in eq. (15), $\Lambda$ is the pumping rate related to current injection. Equations (15) and (16) are now self-consistent with the Maxwell equation, eq. (12). These equations (15), (16), and (12), constitute our set of self-consistent working equations.

If we perform the averaging of eqs. (15) and (16) over the joint density of states, applying the operation defined by eq. (11) to both sides of each equation, we obtain

$$\mathrm{d}W/\mathrm{d}t=-\gamma_L(W-W_{th})$$
$$-(\mathrm{i}\mu/\hbar)(\mathscr{E}^*p-p^*\mathscr{E})+\langle\Lambda\rangle\,, \tag{19}$$

$$\mathrm{d}p/\mathrm{d}t=-\gamma_T(1+\mathrm{i}\Delta)p-\gamma_T\langle\mathrm{i}\delta\rho\rangle$$
$$+(\mu/2\mathrm{i}\hbar)\mathscr{E}W-\mathrm{i}\epsilon pW\,. \tag{20}$$

Here, $W=\langle w\rangle=\langle\rho_{11}-\rho_{22}\rangle$. The term $\gamma_c(w-\bar{w})$ involving the intraband relaxation does not appear

in eq. (19) since $W=\langle \bar{w} \rangle$ simply because the total carrier density is not affected by the redistribution of carriers within the conduction and valence bands. The problem for inhomogeneous effects resides in the second term on the right side of eq. (20). We make use of the perturbative procedure of Graham and Cho [11], to obtain from eqs. (15) and (16) a set of coupled Bloch-like equations which relax the requirement for a detailed treatment of the band structure required due to the presence of the inhomogeneous contribution in eq. (20).

To proceed further, we follow the procedure of Graham and Cho [11], and introduce a parameter $\mathscr{S}^2$ and a new variable $S$, such that

$$\mathscr{S}^2 S = \langle i\delta\rho \rangle . \tag{21}$$

Thus, we eliminate the explicit inhomogeneous contribution from eq. (20), and taking the time derivative of eq. (21), introduce an additional coupled equation of motion for the new variable,

$$\begin{aligned} dS/dt = &-\gamma_T(1+i\Delta)S + (\gamma_T/\mathscr{S}^2)\langle \delta^2\rho \rangle \\ &+ (\mu/2\hbar)\mathscr{E}\langle \delta w \rangle/\mathscr{S}^2 + \epsilon p \langle \delta w \rangle/\mathscr{S}^2 . \end{aligned} \tag{22}$$

Because of this procedure, we have introduced two new variables corresponding to the last three terms in eq. (22), which give rise to two additional coupled equations of motion. Let

$$U = \langle \delta w \rangle/\mathscr{S}^2 , \tag{23}$$

$$p_0 = \langle (1-\delta^2/\mathscr{S}^2)\rho \rangle . \tag{24}$$

It is noted that eq. (24) now involves the second moment in $\delta$. Using eqs. (23) and (24) in eq. (22) and taking the time derivative of eq. (23), we have

$$\begin{aligned} dS/dt = &-\gamma_T(1+i\Delta)S + \gamma_T(p-p_0) \\ &+ (\mu/2\hbar)\mathscr{E}U + \epsilon p U , \end{aligned} \tag{25}$$

$$\begin{aligned} dU/dt = &-\gamma_c(U-\bar{U}) - \gamma_L(U-U_{th}) \\ &- (\mu/\hbar)(\mathscr{E}^* S + \mathscr{E} S^*) \\ &- 2\epsilon(p^* S + S^* p) + \langle \delta\Lambda \rangle/\mathscr{S}^2 , \end{aligned} \tag{26}$$

where $\bar{U}$ is the effective steady-state value [9]. Since $\gamma_c \gg \gamma_L$, this term will give a lowest order contribution near steady state as will become clear later on. It is expected that $U_{th} \approx 0$, since $U$ is nonzero only if the distribution of states is asymmetric (see eq. (23)), and its relaxation is expected to be dominated by intraband relaxation. The value of $\bar{U}$ is sensitive to induce asymmetry.

We must terminate the hierarchy at some point. Here, we will address the situation for first-order contributions, so we terminate the hierarchy at the second moment contribution in the perturbation by setting $p_0=0$. From eq. (24), this gives an effective evaluation of $\mathscr{S}^2$,

$$\mathscr{S}^2 = \langle \delta^2\rho_{ss} \rangle / \langle \rho_{ss} \rangle , \tag{27}$$

where we have defined $\mathscr{S}^2$ in terms of the steady state, $\rho_{ss}$. We have, now, the following four-dimensional set of nonlinear, coupled equations,

$$\begin{aligned} dW/dt = &\langle \Lambda \rangle - \gamma_L(W-W_{th}) \\ &+ (\mu/i\hbar)(\mathscr{E}^* p - p^* \mathscr{E}) , \end{aligned} \tag{28}$$

$$\begin{aligned} dp/dt = &-\gamma_T(1+i\Delta)p - \mathscr{S}^2\gamma_T S \\ &+ (\mu/2i\hbar)\mathscr{E}W - i\epsilon p W , \end{aligned} \tag{29}$$

$$\begin{aligned} dS/dt = &-\gamma_T(1+i\Delta)S + \gamma_T p \\ &+ (\mu/2\hbar)\mathscr{E}U + \epsilon p U , \end{aligned} \tag{30}$$

$$\begin{aligned} dU/dt = &-\gamma_c(U-\bar{U}) - \gamma_L(U-U_{th}) \\ &- (\mu/\hbar)(\mathscr{E}^* S + \mathscr{E} S^*) \\ &- 2\epsilon(p^* S + S^* p) + \langle \delta\Lambda \rangle/\mathscr{S}^2 . \end{aligned} \tag{31}$$

It is noted that eqs. (28) and (29) reduce to the usual optical Bloch equations if we set $\mathscr{S}^2=\epsilon=0$. Equation (30) has a form similar to eq. (29) and is polarization-like, but through two of its terms couples with $p$ to cause counter-phase contributions. Equation (31) has the form of population difference evolution, similar to eq. (28), but couples through $S$ and $p$ to cause changes in $W$. It accounts for the asymmetry in the distribution of electrons (see eq. (23)). All of the details of the band structure are expressed through the parameter $\mathscr{S}^2$, given by eq. (27).

### 3. Rate equation limit

Equations (28)–(31) are certainly rich in their dynamical structure. It is, therefore, interesting to study them by numerical integration, and such a study will be relegated to a future treatment [20]. For the present, however, we focus on the effects which arise within the rate-equation approximation,

i.e., for conditions consistent with adiabatic elimination,

$$|\mathrm{d}p/\mathrm{d}t| \ll \gamma_\mathrm{T} p\,, \quad |\mathrm{d}S/\mathrm{d}t| \ll \gamma_\mathrm{T} S\,. \tag{32}$$

In order to proceed unencumbered, we also set $\langle \delta\Lambda\rangle = 0$ and $\Delta = \epsilon = 0$. We shall address the effects of including $\epsilon$ in the analysis in a future publication [21].

Assuming, in addition, that $\gamma_\mathrm{c} \gg \gamma_\mathrm{L}$, we eliminate $p$, $S$ and $U$ from eq. (28) in the adiabatic limit, $\mathrm{d}p/\mathrm{d}t = \mathrm{d}S/\mathrm{d}t = \mathrm{d}U/\mathrm{d}T = 0$. These conditions result in the relation

$$U = \bar{U}/(1+I)\,, \tag{33}$$

where $I = |\mathscr{E}|^2/\tilde{I}_\mathrm{s}$, $\tilde{I}_\mathrm{s}$ is the intraband saturation intensity given by

$$\tilde{I}_\mathrm{s} = (\hbar^2\gamma_\mathrm{c}\gamma_\mathrm{T}/\mu^2)(1+\mathscr{S}^2)\,, \tag{34}$$

and

$$p = -\frac{\mu\mathscr{E}}{2\hbar\gamma_\mathrm{T}} \frac{\mathrm{i}W + \mathscr{S}^2\bar{U}/(1+I)}{1+\mathscr{S}^2}\,, \tag{35}$$

$$S = -\frac{\mu\mathscr{E}}{2\hbar\gamma_\mathrm{T}} \frac{\mathrm{i}W - \bar{U}/(1+I)}{1+\mathscr{S}^2}\,. \tag{36}$$

If eqs. (33), (35), and (36) are used in eq. (28), the result is

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \langle \Lambda\rangle - \gamma_\mathrm{L}(W - W_\mathrm{th}) - \frac{\mu^2}{\hbar^2\gamma_\mathrm{T}} \frac{|\mathscr{E}|^2 W}{1+\mathscr{S}^2}\,. \tag{37}$$

Writing the gain $\langle \Lambda\rangle$ in terms of the injection current $I_\mathrm{in}$ and $W$ in terms of the carrier density $N$ as

$$N = \tfrac{1}{2}N_\mathrm{t}(W - W_\mathrm{th})\,, \tag{38}$$

equation (37) becomes

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{I_\mathrm{in}}{qV} - \gamma_\mathrm{L}N - \frac{\gamma_\mathrm{L}}{1+\mathscr{S}^2}\frac{|\mathscr{E}|^2}{I_\mathrm{s}}(N - N_0)\,, \tag{39}$$

where $N_0 = -\frac{1}{2}N_\mathrm{t}W_\mathrm{th}$, $I_\mathrm{in} = qVN_\mathrm{t}\langle\Lambda\rangle/2$, and $I_\mathrm{s} = (\hbar^2/\mu^2)\gamma_\mathrm{T}\gamma_\mathrm{L}$ is the interband saturation intensity. Equation (39) is the main result of our paper. This equation is isomorphic to the simplest standard rate equation for semiconductor lasers [6]. In addition, the usual phenomenological gain parameter $a$, introduced by writing the last term as $a(N-N_0)|\mathscr{E}|^2$,

$$a = \frac{\mu^2}{\hbar^2\gamma_\mathrm{T}(1+\mathscr{S}^2)}\,, \tag{40}$$

is determined in terms of the parameters of the two-level, inhomogeneously broadened model, specifically, the dipole moment $\mu$, the dipole relaxation rate $\gamma_\mathrm{T}$, and the inhomogeneous distribution parameter, $\mathscr{S}^2$.

From the consecutive relation, $p = 2\mu N_\mathrm{t} p = \epsilon_0\chi\mathscr{E}$, together with the analytic relaxation for $p$, eq. (35), we obtain the following expression for the susceptibility, $\chi$,

$$\chi = -\frac{\mu^2 N_\mathrm{t}}{\epsilon_0\hbar\gamma_\mathrm{T}(1+\mathscr{S}^2)}\left(\mathrm{i}W + \frac{\mathscr{S}^2\bar{U}}{1+|\mathscr{E}|^2/\tilde{I}_\mathrm{s}}\right). \tag{41}$$

The real and imaginary parts of $\chi$ are related to the index change [22], $\Delta n$, and the optical gain, $g$,

$$g = -2k\,\mathrm{Im}\,\chi = \frac{\mu^2\omega N_\mathrm{t}}{2\epsilon_0\hbar\gamma_\mathrm{T}c(1+\mathscr{S}^2)}\,W\,, \tag{42}$$

$$\Delta n = \frac{\mathrm{Re}\,\chi}{2n} = -\frac{\mu^2\mathscr{S}^2\bar{U}N_\mathrm{t}}{\epsilon_0 n\hbar\gamma_\mathrm{T}(1+\mathscr{S}^2)}\frac{1}{1+|\mathscr{E}|^2/\tilde{I}_\mathrm{s}}\,. \tag{43}$$

The linewidth enhancement factor [22], $\beta_\mathrm{c}$, is defined as the ratio $\mathrm{Re}\,\chi/\mathrm{Im}\,\chi$ and is given by

$$\beta_\mathrm{c} = \frac{\mathrm{Re}\,\chi}{\mathrm{Im}\,\chi} = \frac{\mathscr{S}^2(\bar{U}/W)}{1+|\mathscr{E}|^2/I_\mathrm{s}}\,. \tag{44}$$

It is noted that $\beta_\mathrm{c}$ is not a constant, but depends upon the inversion $W$ as well as the intensity. Its origin lies in the asymmetric nature of the inversion in semiconductors through the parameter $\bar{U}$ (see eq. (23)). For a two-level atomic system $\beta_\mathrm{c} = 0$ at resonance simply because $\bar{U} = \langle\delta w\rangle/\mathscr{S}^2$ is zero. The intensity dependence of $\beta_\mathrm{c}$ has its origin in the field-induced changes in $\bar{U}$ through redistribution of electrons within the conduction band.

## 4. Conclusion

We have derived a four-dimensional Bloch-like set of coupled equations for an inhomogeneously broadened, two-level system interacting via dipole–dipole interactions as well as with the electromagnetic field, eqs. (28)–(31), which are coupled, self-consistently, with the Maxwell field equation, eq. (12). We neglected detuning, $\Delta$, and near dipole–dipole interactions to show, for the first time, that a two-level model reduces, in the rate equation limit, to the usual rate equation for the carrier density, eq. (39), where

the usual phenomenological gain parameter, $a$, eq. (40), is determined in terms of the two-level model parameters and the field intensity. We have furthermore, obtained expressions for the gain, $g$, and index change, $\Delta n$, eqs. (42) and (43), respectively, in terms of system parameters, and we have derived an explicit expression for the linewidth enhancement factor, $\beta_c$, eq. (44).

The obvious agreement of our results, in the rate equation limit, with the standard relations, implies that the four-dimensional set of equations, eqs. (28)–(31), and (12), should be very important for predictions for pulse propagation, phase modulation, and dynamical instabilities. Although the approximation to steady state for the perturbation parameter, $\mathscr{S}^2$, eq. (27), should be quite adequate in the rate-equation regime, this approximation in relation to the four-dimensional model has been shown by Meziane [23] to lead to significant departures from the Monte Carlo results for large dynamical departures from steady-state. That author found significant improvement and remarkably close agreement when $\mathscr{S}^2$, eq. (27), was replaced, arbitrarily, by the saturable relation, $\mathscr{S}^2 \to \mathscr{S}^2/(1+|\mathscr{E}|^2)$. Since all of the band structure detail in this formulation is contained in $\mathscr{S}^2$, a knowledge of the structure of the joint density of states may allow a better approximation, and, in particular, lead to a self-consistent functional dependence. For instance, the steady-state solution of eqs. (15) and (16) when $\epsilon=0$ is given by

$$\rho_{ss} = \frac{u}{2\hbar\gamma_T}\frac{\mathscr{E} w_{ss}}{1+i\delta}, \tag{45}$$

$$w_{ss} = \frac{\bar{w}(1+\delta^2)}{1+\delta^2+|\mathscr{E}|^2/\tilde{I}_s}. \tag{46}$$

One might use eqs. (45) and (46), together with some assumptions about the joint density of states, to actually integrate the expression in eq. (27) to yield $\mathscr{S}^2 = \mathscr{S}^2(|\mathscr{E}|^2)$ in a self-consistent manner, within the structure of the model and a particular material system. Further aspects of these ideas will be explored in the near future.

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## References

[1] M. Lindberg and S.W. Koch, Phys. Rev. B 38 (1988) 3342; S.W. Koch, N. Peyghambarian and M. Lindberg, J. Phys. C 21 (1988) 5229;
M. Sargent III, F. Zhou, S. An, M. Lindberg and S.W. Koch, in: Quantum optics V, Springer Proc. Phys. 41, eds. J.D. Harvey and D.F. Walls (Springer, Berlin, 1989).
[2] C. Flytzanis and J.L. Oudar, eds., Nonlinear optics: materials and devices, Springer Proc. Phys. 7 (Springer, Berlin, 1986).
[3] L. Allen and J.H. Eberly, Optical resonance and two-level atoms (Wiley, New York, 1975).
[4] P. Meystre and M. Sargent III, Quantum optics (Springer, Berlin, 1990).
[5] G.P. Agrawal and N.K. Dutta, Long-wavelength semiconductor lasers (Van Nostrand Reinhold, New York, 1986).
[6] M. Yamada and Y. Suematsu, J. Appl. Phys. 52 (1981) 2653; IEEE J. Quantum Electron. QE-21 (1985) 434.
[7] G.P. Agrawal, J. Opt. Soc. Am. B 5 (1988) 147.
[8] B. Thedrez, A. Jones and R. Frey, J. Quantum Electron. 24 (1988) 1499;
F. De Rougemont and R. Frey, Phys. Rev. B 37 (1988) 1237.
[9] G.P. Agrawal, IEEE J. Quantum Electron. QE-23 (1987) 8601.
[10] G.P. Agrawal, J. Appl. Phys. 63 (1988) 1232.
[11] R. Graham and Y. Cho, Optics Comm. 47 (1983) 52.
[12] J. Van Kranendonk and J.E. Sipe, in: Progress in Optics XV, ed. E. Wolf (North-Holland, Amsterdam, 1977) p. 245.
[13] Y. Ben-Aryeh, C.M. Bowden and J.C. Englund, Phys. Rev. A 34 (1986) 3917.
[14] C.M. Bowden, Near dipole–dipole interaction effects in quantum and nonlinear optics, Intern. Conf. on Quantum Optics, Hyderabad, India, 1991 (Proceedings, Plenum, NY, 1992) in press.
[15] C.M. Bowden and J.P. Dowling, Phys. Rev. A 47 (1993) 1247.
[16] R. Friedberg, S.R. Hartmann and J.T. Manassah, Phys. Rep. 7 (1973) 101.
[17] J.D. Jackson, Classical electrodynamics, 2nd Ed. (Wiley, New York, 1975) Ch. 4.
[18] C. Kittel, Introduction to solid state physics, 6th Ed. (Wiley, New York, 1986).
[19] J.J. Maki, M.S. Malcuit, J.E. Sipe and R.W. Boyd, Phys. Rev. Lett. 67 (1991) 972.
[20] C.M. Bowden, G.P. Agrawal and S. Singh, to be published.
[21] G.P. Agrawal and C.M. Bowden, to be published.
[22] G.P. Agrawal, IEEE J. Quantum Electron. 26 (1990) 1901.
[23] B. Meziane, Optics Comm. 75 (1990) 287.