Effect of phase-conjugate feedback on semiconductor laser dynamics

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The effect of phase-conjugate feedback on the dynamic response of semiconductor lasers is studied by using a rate-equation approach. The steady state exists only for certain well-defined values of the phase of the intracavity optical field. Depending on the amount of phase-conjugate feedback, the steady state becomes unstable through two independent instabilities, referred to as fold and Hopf instabilities. The fold instability is due solely to the phase-conjugate nature of the feedback and does not occur in the case of normal feedback. In the instability region, the laser output is found to become chaotic by following a period-doubling or quasi-periodic route to chaos, depending on the amount of feedback.

Semiconductor lasers are known to be extremely sensitive to external optical feedback.[1-8] Depending on the amount and the phase of feedback, the feedback may improve the laser characteristics (such as a reduced linewidth[4]) or may degrade the laser (e.g., enhanced intensity noise[5]). It can even destabilize the laser and lead to optical chaos.[6-8] In general, the laser dynamics depend on the phase shift acquired in the external cavity. This dependence on the external phase shift can be eliminated if the feedback occurs from a phase-conjugate mirror (PCM) placed at a distance from the semiconductor laser, simply because the phase shift in the forward direction is canceled during the backward trip from the PCM to the laser. The objective of this Letter is to investigate the effect of phase-conjugate feedback on laser dynamics through the well-known rate equations.[1] Even though the coupling of a PCM to semiconductor lasers has been considered previously,[9-14] the dynamic aspects have attracted little attention.

The mathematical model follows closely the case of ordinary optical feedback described in Sec. 6.7 of Ref. 1. The only difference in the rate equations occurs in the feedback term, which should include the phase-conjugate nature of the feedback by changing the optical field E to E^*. The resulting rate equations are[1]

$$\dot{E}(t) = i(\omega_0 - \Omega)E(t) + \frac{1}{2}(G - \gamma)(1 - i\alpha)E(t) + \kappa E^*(t - \tau)\exp(i\phi_{\mathrm{PCM}}), \quad (1)$$

$$\dot{N}(t) = I/q - \gamma_e N(t) - G|E(t)|^2, \quad (2)$$

where the dot represents a derivative with respect to time. Various parameters have their usual meaning.[1] Specifically, ω_0 is the optical frequency with feedback, while Ω is its value in the absence of feedback. $G = G_N(N - N_0)$ is the optical gain assumed to vary linearly with the electron population N, γ is the cavity decay rate related to the photon lifetime $\tau_p = \gamma^{-1}$, α is the linewidth enhancement factor with typical values in the range of 4–8, I is the injection current, and γ_e is the population decay rate related to the electron lifetime $\tau_e = \gamma_e^{-1}$. The optical field E is normalized such that $|E|^2$ represents the intracavity photon number P.

The feedback term in Eq. (1) consists of three parameters, κ, τ, and ϕ_{PCM}. The feedback rate κ and the round-trip time τ are given by

$$\kappa = \frac{\eta_c(1 - R_m)}{\tau_L}\left(\frac{R_{\mathrm{PCM}}}{R_m}\right)^{1/2}, \qquad \tau = \frac{2L_{\mathrm{ext}}}{c}, \quad (3)$$

where η_c is the coupling loss, R_m is the laser facet reflectivity, τ_L is the round-trip time in the laser cavity, R_{PCM} is the reflectivity of the PCM, and L_{ext} is the spacing between the laser and the PCM. The parameter ϕ_{PCM} is a constant phase shift occurring at the PCM. It is included here for generality. The extra phase shift $\omega_0\tau$ occurring for ordinary feedback is absent in Eq. (1) because of the phase-conjugate nature of the PCM. Note, however, that even though the steady-state solutions of Eqs. (1) and (2) do not depend on the PCM location (because of the absence of $\omega_0\tau$ phase shift), the dynamic behavior still depends on it because of a delayed feedback. Finally, the PCM is assumed to respond instantaneously in Eq. (1). If the PCM responds slower than the round-trip time τ, κ would become time dependent. This case can be studied by adding a third equation that governs the PCM dynamics.

The steady-state solution of Eqs. (1) and (2) is obtained by setting the time derivatives to zero. The result is

$$G = \gamma - 2\kappa\cos\theta, \quad (4)$$

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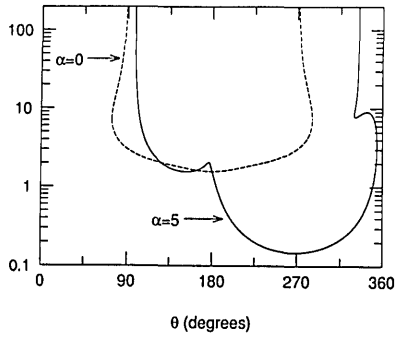

Fig. 1. Hopf-instability domain for $\alpha = 0$ and 5 in the parameter space formed by $\kappa\tau$ and θ. The steady-state solution is unstable inside the region of each curve.

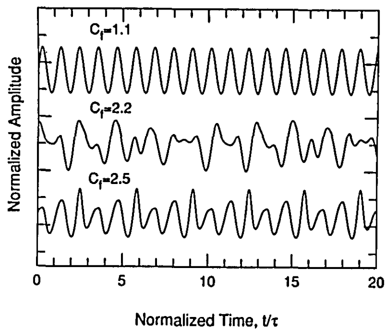

Fig. 2. Pulse trains emitted by the laser for $C_f = 1.1$, 2.2, and 2.5 showing the dynamic evolution of the output from self-pulsing (period 1) to chaos and then to period 3 as the amount of phase-conjugate feedback increases.

$$\omega_0 = \Omega - \kappa(\sin\theta + \alpha\cos\theta), \tag{5}$$

where

$$\theta = 2\phi + \phi_{\mathrm{PCM}} \tag{6}$$

and ϕ is the optical phase defined by $E = \sqrt{P}\exp(-i\phi)$. Thus the effect of feedback is to change the threshold gain and the optical frequency from their solitary-laser values γ and Ω. The threshold is reduced for $|\theta| < \pi/2$; maximum reduction occurs for $\theta = 0$. In that case, the phase ϕ is pinned to a value $\phi = -\phi_{\mathrm{PCM}}/2$ (i.e., the laser phase is governed by the PCM). The situation is different from the case of normal feedback for which the laser phase remains arbitrary. Another major difference is related to the laser frequency. Equation (5) has a single solution for all values of κ, whereas multiple solutions are allowed in the case of normal feedback. These multiple solutions correspond to the longitudinal modes of external cavity. Since the steady-state solution of Eqs. (1) and (2) does not depend on the PCM location, external-cavity modes play no role; the only effect of phase-conjugate feedback is to shift the laser frequency slightly. The maximum frequency shift of $\kappa(1 + \alpha^2)^{1/2}$ occurs for $\theta = -\tan^{-1}\alpha$.

Since the value of θ remains undetermined in Eqs. (4) and (5), a natural question is how the stability of the steady state depends on θ. To investigate the stability issue, we perform a linear-stability analysis of Eqs. (1) and (2) by linearizing them in small fluctuations around the steady-state values. Since the procedure is straightforward and has been discussed previously,[1-3] only a brief description is given here. The three linearized equations corresponding to fluctuations in P, ϕ, and N are solved by assuming a time dependence of the form $\exp(zt)$, where z is the growth rate of perturbation. It is determined by solving an eigenvalue equation of the form

$$\begin{aligned}&[(z + \Gamma_P)(z + \Gamma_N) + GG_NP + (z + \Gamma_N)\\ &\quad\times \kappa\cos\theta(1 - e^{-z\tau})][z + \kappa\cos\theta(1 + e^{-z\tau})]\\ &\quad= \kappa\sin\theta[\alpha GG_NP - (z + \Gamma_N)\\ &\qquad\times \kappa\sin\theta(1 - e^{-z\tau})](1 + e^{-z\tau}),\end{aligned} \tag{7}$$

where

$$\Gamma_P = R_{\mathrm{sp}}/P + \epsilon_{\mathrm{NL}}GP,$$
$$\Gamma_N = \gamma_e + N(\partial\gamma_e/\partial N) + G_NP. \tag{8}$$

In Eqs. (8), R_{sp} is the spontaneous emission rate and ϵ_{NL} is the nonlinear-gain parameter ($\epsilon_{\mathrm{NL}} \sim 10^{-7}$) introduced phenomenologically.[1] The eigenvalue equation, Eq. (7), is solved numerically to find the growth rate z. The steady-state solution becomes unstable whenever the real part of z is positive since small fluctuations grow exponentially in that case.

In the absence of feedback ($\kappa = 0$), Eq. (7) corresponds to a solitary laser and is readily solved. It has three solutions, one of them being $z = 0$. The other two are given by $z = -\Gamma_R \pm i\Omega_R$, where

$$\Omega_R = [GG_NP - (\Gamma_P - \Gamma_N)^2/4]^{1/2},$$
$$\Gamma_R = (\Gamma_P + \Gamma_N)/2. \tag{9}$$

Since the real part of z is never positive, the solitary laser is stable. Small fluctuations decay toward the steady state through relaxation oscillations; Ω_R and Γ_R represent the frequency and the damping rate of relaxation oscillations.[1]

In the presence of feedback, both the real root and the pair of complex conjugate roots change their values. The real root becomes positive for all values of $\kappa\tau$ when θ lies in the range of θ_c to $\pi + \theta_c$, where θ_c

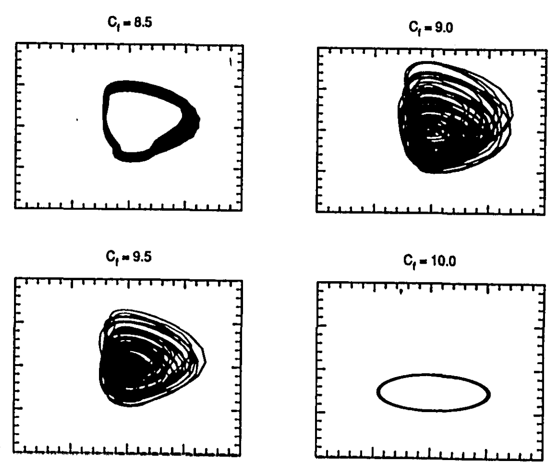

Fig. 3. Phase portraits (P versus N) for $C_f = 8.5$, 9, 9.5, and 10 showing the quasi-periodic route to chaos in the Hopf-instability region.

is a critical value given by

$$\tan\theta_c = \frac{1}{\alpha}\left(1 + \frac{\Gamma_P\Gamma_N}{GG_NP}\right) \approx \frac{1}{\alpha}. \qquad (10)$$

For $\alpha = 0$, $\theta_c = 90°$, and the steady-state solution is then unstable for values of θ such that $90° < \theta < 270°$. For $\alpha = 5$, $\theta_c = 11.3°$, and the instability region corresponds to values of θ in the range of 11.3° to 191.3°. Since the eigenvalue crosses the complex $z = 0$ plane along the real axis, we refer to this instability as the fold instability.[15] It is interesting to note that the fold instability never occurs in the case of normal feedback, since the eigenvalue equation does not have a real solution such that $z > 0$.

The steady-state solution can also become unstable if the real part of the pair of complex roots becomes positive. Such an instability is referred to as the Hopf instability[15] and corresponds to the situation in which the external feedback changes the damping rate Γ_R in such a way that relaxation oscillations are no longer damped. Figure 1 shows the instability region in the plane formed by the parameters $\kappa\tau$ and θ for $\alpha = 0$ and 5. The calculations were done by using typical parameter values[1] for an index-guided InGaAsP laser operating at 2 mW of power in the absence of feedback. Specifically, $\Omega_R/2\pi = 2.65$ GHz, $\Gamma_N = 1.27$ GHz, and $\Gamma_P = 2.56$ GHz. The external cavity is taken to be 5 cm long with a round-trip time $\tau = 0.33$ ns. The steady state is unstable inside the region bounded by the curve for which $\mathrm{Re}(z) = 0$. A notable feature of Fig. 1 is that the Hopf instability exists even for $\alpha = 0$, although the instability region changes considerably with α.

A natural question is what happens to the laser output in the instability region in which the laser cannot operate continuously even at a constant applied current. To answer this question, we have solved Eqs. (1) and (2) numerically. The laser output exhibits a rich variety of dynamical features depending on the amount of feedback. Figure 2 shows the emitted pulse train for three values of the feedback parameter $C_f = \kappa\tau(1 + \alpha^2)^{1/2}$ and for $\theta = 150°$ and $\alpha = 5$. The other parameters remain the same. For $C_f = 1.1$, the output is periodic with a repetition rate close to the relaxation-oscillation frequency of the solitary laser (2.65 GHz). Self-pulsing can therefore be interpreted as undamped relaxation oscillations. The periodic solution becomes unstable when C_f exceeds 1.6, and the laser output becomes chaotic, following a period-doubling route. Figure 2 shows the chaotic output for $C_f = 2.2$. Chaos gives way to a period-3 window beyond a critical value of C_f close to 2.4. The period-3 time series is also shown in Fig. 2 for $C_f = 2.5$. The laser returns to a regular, period-1 self-pulsing for $C_f = 2.9$, but with a repetition rate of approximately 3.2 GHz.

The period-doubling route to chaos discussed above corresponds to the fold instability as it is outside the Hopf-instability region shown in Fig. 1. For $\theta = 150°$, Hopf instability occurs only for $C_f > 7.5$. One would expect qualitatively new dynamic behavior when both fold and Hopf instabilities occur simultaneously. Our numerical simulations show that this is indeed the case. The laser output is found to become chaotic after following a quasi-periodic route to chaos for $C_f > 7.5$. Figure 3 shows the phase diagrams (P versus N) for $C_f = 8.5$, 9, 9.5, and 10. The quasi-periodic dynamics of the laser at $C_f = 8.5$ gives way to chaos as C_f increases. The system returns to the quasi-periodic state for $C_f = 10$, but at a much higher repetition rate. The repetition rate is close to the solitary-laser relaxation-oscillation frequency (2.65 GHz) for $C_f = 8.5$ but nearly triples for $C_f = 10$. Chaos develops again as C_f increases beyond 10. There appear to exist multiple chaotic windows in the Hopf-instability region shown in Fig. 1. A quasi-periodic route to chaos occurs for other values of θ also.

In conclusion, we have studied the effect of phase-conjugate feedback on the semiconductor-laser dynamics. The steady state exists only for certain well-defined values of the phase of the optical field inside the laser cavity. The steady state becomes unstable through two independent instabilities, referred to here as fold and Hopf instabilities. The fold instability is due solely to the phase-conjugate nature of the feedback and does not occur in the case of normal feedback.

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