Effect of Gain Dispersion on Ultrashort Pulse Amplification in Semiconductor Laser Amplifiers

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Abstract—The effect of gain dispersion in semiconductor laser amplifiers is discussed by considering an amplifier model that includes both gain saturation and gain dispersion. The saturated and unsaturated amplification regimes, corresponding to whether the pulse energy is comparable to or much smaller than the saturation energy, are discussed separately. In the unsaturated regime, gain-induced group-velocity dispersion is found to play an important role. The amplified pulse can be compressed or broadened depending on whether the input pulse is chirped or unchirped. The pulse spectrum remains largely unaffected in the linear regime. In the saturation regime, self-phase modulation leads to considerable changes in the pulse spectrum. When both gain saturation and gain dispersion act together, the shape and the spectrum of the amplified pulse develop new features which depend on the shape, the width, and the frequency chirp of the input pulse.

I. INTRODUCTION

SEMICONDUCTOR laser amplifiers have attracted considerable attention [1]-[11] in recent years as they are capable of providing high single-pass gain (~ 30 dB) over a wide bandwidth (~ 5 THz) in the form of a compact and efficient device. Because of a large bandwidth, they can amplify optical pulses as short as a few picoseconds [5]-[9] without a significant change in the pulse width. The pulse spectrum, however, changes considerably when the amplifier operates in the saturation regime [7]-[10]. Spectral changes occur as a result of self-phase modulation (SPM), a nonlinear phenomenon that occurs whenever the refractive index becomes intensity dependent. Such an intensity dependence of the refractive index can occur in semiconductor laser amplifiers because of an amplitude-phase coupling governed by the linewidth enhancement factor [8], [9]. More explicitly, intensity-induced changes in the carrier density occurring because of gain saturation change the refractive index of the active medium and lead to SPM.

For optical pulses shorter than a few picoseconds, it is necessary to include the effects of a finite gain bandwidth. The spectrum of such ultrashort pulses is wide enough so

that all spectral components cannot be amplified by the same amount because of the gain rolloff occurring at high frequencies. This phenomenon is generally referred to as "gain dispersion." Because of an interrelationship between the gain and the refractive index, governed by the linewidth enhancement factor, one would expect that gain dispersion would be accompanied by group-velocity dispersion (GVD). Such gain-induced GVD can affect the pulse characteristics significantly during amplification. The objective of this paper is to study the role of gain dispersion and the associated GVD on pulse amplification in semiconductor laser amplifiers. The basic model is developed in Section II and includes the effects of gain dispersion, gain saturation, and the gain-induced GVD and SPM. Section III considers the unsaturated amplification regime for weak pulses such that the pulse energy remains well below the saturation energy of the amplifier (~ 5-10 pJ). The effects of gain saturation are discussed in Section IV with particular emphasis on the spectral changes occurring as a result of gain-induced SPM. Finally, the results are summarized in Section V. The main conclusion is that the carrier-induced GVD can lead to pulse compression under certain conditions, even when the amplifier operates in the unsaturation regime. In the saturation regime, it can also affect the pulse spectrum through SPM.

II. BASIC EQUATIONS

The theory of pulse propagation in semiconductor laser amplifiers has been considered before [9]. The amplifier is modeled as a traveling-wave amplifier to avoid a complicated boundary-value problem. The electric field associated with the optical pulse is written in the form

$$E(x, y, z, t) = \frac{1}{2} \{\hat{x} F(x, y) A(z, t)$$
$$\cdot \exp [i(k_0 z - \omega_0 t)] + \text{c.c.}\}, \quad (1)$$

where $\hat{x}$ is the polarization unit vector, $F(x, y)$ is the transverse profile of the laser assumed to oscillate in a single mode, $k_0 = \bar{n} \omega_0/c$, $\bar{n}$ is the effective mode index, ω_0 is the optical frequency, and $A(z, t)$ is the slowly varying pulse envelope. By substituting (1) in the Maxwell's wave equation, $A(z, t)$ is found to satisfy the basic pulse-prop-

agation equation [9]

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2} = \frac{1}{2}(1 - i\alpha)gA - \frac{1}{2}\alpha_{\text{int}}A$$

$$(2)$$

where v_g is the group velocity, β_2 is the material dispersion parameter, g is the amplifier gain, α_{int} is the internal waveguide loss, and α is the linewidth enhancement factor. The gain-induced index changes are governed by the parameter α. In particular, gain saturation occurring at high intensities leads to intensity-dependent index changes responsible for SPM and for spectral broadening of the amplified pulses [7]-[9].

In general, the optical gain g in (2) is time dependent because of two physical phenomena known as gain saturation and gain dispersion. Gain saturation occurs for optical pulses whose input energy is comparable to the saturation energy of the amplifier (typically 5–10 pJ). On the other hand, gain dispersion occurs for optical pulses whose input width is short enough so that their spectral width becomes comparable to the gain bandwidth. The effect of gain dispersion is to reduce the gain for spectral components far away from the carrier frequency ω_0. Gain dispersion can be included by considering the gain spectral profile $\tilde{g}(\omega)$, where tilde over g is a reminder that the gain is being considered in the frequency domain. For input pulses of width larger than the intraband relaxation time, we can expand $\tilde{g}(\omega)$ about ω_0 in a Taylor series. By retaining the terms up to second order in $\omega - \omega_0$, we obtain

$$\tilde{g}(\omega) = \tilde{g}(\omega_0) + g'(\omega - \omega_0) + \frac{1}{2}g''(\omega - \omega_0)^2 \quad (3)$$

where g' and g'' are the first and second derivatives of $\tilde{g}$ evaluated at ω_0. For an optical pulse whose carrier frequency ω_0 coincides with the gain peak, $g' = 0$. For simplicity, we assume this to be the case; the analysis can be easily extended to include the case $g' \neq 0$. The term $\tilde{g}(\omega_0)$ then equals the peak value g_p of the gain. The parameter g'' is related to the curvature of the gain profile at the gain peak and is generally negative. For notational simplicity, we write $g'' = -2g_p T_2^2$, where T_2 is analogous to the polarization relaxation time of a two-level system and is related to the intraband relaxation time τ_{in}. Typically, $T_2 \sim 0.1$ ps for semiconductor lasers. Equation (3) can thus be written as

$$\tilde{g}(\omega) = g_p[1 - T_2^2(\omega - \omega_0)^2].$$

In the time-domain description, $\omega - \omega_0$ is replaced by the operator $i(\partial/\partial t)$. The quantity gA in (2) is then given by

$$gA = g_p A + g_p T_2^2 \frac{\partial^2 A}{\partial t^2} \quad (4)$$

where the effects of gain dispersion are governed by the last term. The parameter T_2 is a measure of the importance of gain dispersion. In particular, gain dispersion is absent for $T_2 = 0$.

The time dependence of the peak gain results from gain saturation. In a simple model [12], g_p is assumed to vary linearly with the carrier density N, i.e.,

$$g_p = \Gamma a(N - N_0) \quad (5)$$

where Γ is the confinement factor, a is the gain coefficient, and N_0 is the transparency value of the carrier density. The temporal variation of N is governed by the carrier-density rate equation

$$\frac{\partial N}{\partial t} = \frac{I}{qV} - \frac{N}{\tau_c} - \frac{g_p|A|^2}{\hbar\omega_0\Gamma} \quad (6)$$

where I is the injection current, V is the active volume, and τ_c is the carrier lifetime. Equations (5) and (6) can be combined to obtain

$$\frac{\partial g_p}{\partial t} = \frac{g_0 - g_p}{\tau_c} - \frac{g_p|A|^2}{E_{\text{sat}}} \quad (7)$$

where

$$g_0 = \Gamma a N_0(I/I_0 - 1) \quad (8)$$

with $I_0 = qVN_0/\tau_c$ is the small-signal (unsaturated) gain and

$$E_{\text{sat}} = \hbar\omega_0\sigma/a \quad (9)$$

is the saturation energy of the amplifier (σ is the mode cross section). In (7), A has units such that $|A|^2$ represents the power.

Equations (2), (4), and (7) govern pulse propagation in semiconductor laser amplifiers under quite general conditions. They can be considerably simplified in the case of short optical pulses of width $T_0 \ll \tau_c$. Since $\tau_c = 0.2$–1 ns, this condition is satisfied for $T_0 < 10$ ps. For such short pulses, the first term on the right side of (7) can be ignored. Equation (7) is then easily solved to yield

$$g_p(t) = g_0 \exp\left[-\int_{-\infty}^{t} (|A(z, t)|^2/E_{\text{sat}})\, dt\right]. \quad (10)$$

By substituting (4) and (10) in (2), we obtain the propagation equation

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2} + \frac{1}{2}\alpha_{\text{int}}A$$

$$= \frac{1}{2}(1 - i\alpha)g_0 \exp\left[-\frac{1}{E_{\text{sat}}}\int_{-\infty}^{t} |A(z, t)|^2\, dt\right]$$

$$\cdot \left(A + T_2^2\frac{\partial^2 A}{\partial t^2}\right). \quad (11)$$

Equation (11) is the basic equation of this paper. We explore its implications for pulse amplification in the following sections. It is useful to rewrite (11) in a normalized form by defining

$$U = A/\sqrt{P_0}$$

$$\tau = (t - z/v_g)/T_0 \quad (12)$$

where P_0 is the peak power, T_0 is the width of the input pulse, and τ is the reduced time measured in a frame mov-

ing with the pulse. The normalized propagation equation is given by

$$\frac{\partial U}{\partial z} + \frac{i}{2}\frac{\beta_2}{T_0^2}\frac{\partial^2 U}{\partial \tau^2} + \frac{1}{2}\alpha_{\text{int}}U$$

$$= \frac{1}{2}(1 - i\alpha)g_0 \exp\left[-s\int_{-\infty}^{\tau}|U|^2\,d\tau\right]$$

$$\cdot\left(U + d^2\frac{\partial^2 U}{\partial \tau^2}\right) \qquad (13)$$

where s is related to the input pulse energy E_{in} as

$$s = P_0 T_0/E_{\text{sat}} = (E_{\text{in}}/E_{\text{sat}})\left(\int_{-\infty}^{\infty}|U|^2\,d\tau\right)^{-1} \qquad (14)$$

and d is related to the parameter T_2 as

$$d = T_2/T_0. \qquad (15)$$

Equation (13) is valid only for $d \ll 1$ because of the use of the rate-equation approximation and a parabolic gain profile in its derivation. In the numerical results presented in this paper, the maximum value of d is limited to 0.2 as the model becomes questionable for larger values. Note also that the frequency at which the gain peaks is assumed to be independent of the carrier density. This is a simplification as the gain peak in semiconductor lasers generally shifts to the high-frequency side with an increase in the carrier density.

The parameters s and d in (13) govern the importance of gain saturation and gain dispersion, respectively. When both of them are zero, the amplifier is in the linear regime such that both the pulse shape and pulse spectrum are unaffected by it. When $s = 0$ but $d \neq 0$, the amplifier is in the linear regime, but gain dispersion is expected to lead to spectral narrowing. When $d = 0$ but $s \neq 0$, the amplifier operates in the nonlinear (saturated) regime. Gain saturation then leads to mainly spectral changes as a result of SPM [8], [9]. When both of them are nonzero, the simultaneous presence of gain dispersion and gain saturation can lead to new temporal and spectral features. Before considering this general case, we first consider the case $s = 0$ to see how gain dispersion affects the pulse characteristics in the linear (unsaturated) regime.

III. GAIN-INDUCED GROUP-VELOCITY DISPERSION

In this section, we study how gain dispersion can lead to GVD when the linewidth enhancement factor $\alpha \neq 0$. The effects of GVD can be understood most easily in the linear regime by assuming that the pulse energy remains considerably below the saturation energy during amplification so that $s \ll 1$ and can be set to zero in (13). For simplicity, we neglect the internal loss by setting $\alpha_{\text{int}} = 0$. We also set $\beta_2 = 0$ as material dispersion is negligible under typical operating conditions. Equation (13) can then be written as

$$\frac{\partial U}{\partial z} - \frac{1}{2}(1 - i\alpha)g_0 d^2\frac{\partial^2 U}{\partial \tau^2} = \frac{1}{2}(1 - i\alpha)g_0 U. \qquad (16)$$

This linear equation can be solved exactly by using the Fourier method. The general solution is given by

$$U(z, \tau) = \exp\left[\tfrac{1}{2}(1 - i\alpha)g_0 z\right]\int_{-\infty}^{\infty}\tilde{U}(0, \omega)$$

$$\cdot\exp\left[-\tfrac{1}{2}(1 - i\alpha)d^2 g_0 z\omega^2 - i\omega\tau\right]\,d\omega \qquad (17)$$

where $\tilde{U}(0, \omega)$ is the Fourier transform of the input amplitude $U(0, \tau)$. Consider, for example, a Gaussian input pulse with amplitude

$$U(0, \tau) = \exp(-\tau^2/2). \qquad (18)$$

The amplitude of the amplified pulse at $z = L$, where L is the amplifier length, is then given by

$$U(L, \tau) = \frac{\exp[(1 - i\alpha)g_0 L/2]}{[1 + d^2 g_0 L(1 - i\alpha)]^{1/2}}$$

$$\cdot\exp\left(-\frac{\tau^2/2}{1 + d^2 g_0 L(1 - i\alpha)}\right). \qquad (19)$$

In the absence of gain dispersion ($d = 0$), the input pulse energy is amplified by a factor $G_0 = \exp(g_0 L)$, and the pulse shape and width remain unchanged. For $d \neq 0$, the pulse shape still remains Gaussian, but its width increases (pulse broadening) and it acquires frequency chirp. The broadening factor f_B and the chirp $\Delta\nu$ are given by

$$f_B = [(1 + d^2 g_0 L)^2 + (\alpha d^2 g_0 L)^2]^{1/2}(1 + d^2 g_0 L)^{-1/2} \qquad (20)$$

$$\Delta\nu = -\frac{1}{2\pi}\frac{d\phi}{dt} = \frac{\alpha(d^2 g_0 L)\tau/(2\pi T_0)}{(1 + d^2 g_0 L)^2 + (\alpha d^2 g_0 L)^2}. \qquad (21)$$

Equations (20) and (21) show the effect of linewidth enhancement factor on pulse propagation in the presence of gain dispersion. When $\alpha = 0$, the chirp is absent, and the input pulse broadens by a factor $f_B = (1 + d^2 g_0 L)^{1/2}$. Even for $d = 0.2$ and $G_0 = \exp(g_0 L) = 1000$ (30-dB gain), $f_B = 1.13$, indicating that pulse broadening is only 13%. However, the pulse width almost doubles ($f_B = 1.86$) if we choose $\alpha = 6$, which is a typical value for 1.55 μm InGaAsP amplifiers.

The enhanced broadening of the amplified pulse can be attributed to GVD induced by gain dispersion. One can define a dispersion length as

$$L_D = \frac{1}{\alpha d^2 g_0} = \frac{T_0^2}{\beta_2^{\text{eff}}} \qquad (22)$$

where $\beta_2^{\text{eff}} = \alpha g_0 T_2^2$ is the effective GVD parameter introduced in a manner analogous to that of optical fibers [13]. The effect of material dispersion, neglected in the above analysis, can be included by adding the corresponding β_2

to β_2^{eff}. For $\alpha = 6$, $T_2 = 0.1$ ps, and $g_0 = 300$ cm^{-1}, $\beta_2^{eff} = 18$ ps^2/cm. This value is large enough so that material dispersion is generally negligible. The effects of GVD become important for an amplifier of length $L \sim L_D$. For $T_0 = 0.6$ ps, corresponding to a full width at half maximum (FWHM) of about 1 ps, $L_D \simeq 200$ μm. This estimate shows that the gain-induced GVD effects are expected to become important in practical amplifiers ($L = 250$–500 μm) for input pulses as wide as a few picoseconds. It should be noted that the linewidth enhancement factor α is assumed to be frequency independent in obtaining (22). This is a reasonable approximation as α is not expected to vary significantly over the spectral width of an optical pulse as short as 1 ps. In view of the Taylor expansion (3), the parameter α in (22) corresponds to the value of α at the carrier frequency of the pulse.

As seen in (21), the gain-induced GVD also imposes a frequency chirp on the amplified pulse. However, the pulse spectrum actually becomes narrower than the input spectrum. Spectral narrowing is easily understood by noting that spectral wings are amplified less than the central frequencies associated with the input pulse. At the same time, the energy gain is reduced as a result of nonuniform amplification of different spectral components. The energy gain can be obtained by integrating (19) over τ and is given by

$$G_E = \frac{E_{out}}{E_{in}} = \frac{\int_{-\infty}^{\infty} |U(L, \tau)|^2 \, d\tau}{\int_{-\infty}^{\infty} |U(0, \tau)|^2 \, d\tau} = \frac{G_0}{(1 + d^2 g_0 L)^{1/2}}$$

(23)

where $G_0 = \exp(g_0 L)$ is the energy gain in the absence of gain dispersion ($d = 0$). As an example, the energy gain is reduced by 13% for $G_0 = 30$ dB and $d = 0.2$. Note that the gain reduction does not depend on the linewidth enhancement factor α as one may also expect on physical grounds.

The preceding discussion assumes that the input Gaussian pulse is unchirped. The effect of an initial linear chirp can be included by replacing (18) with

$$U(0, \tau) = \exp[-(1 + iC)\tau^2/2]$$ (24)

where C is the chirp parameter [13] defined such that the frequency increases along the pulse for positive values of C. The integral in (17) can still be performed analytically with the result

$$U(L, \tau) = \frac{\exp[(1 - i\alpha)g_0 L/2]}{[1 + d^2 g_0 L(1 - i\alpha)(1 + iC)]^{1/2}}$$
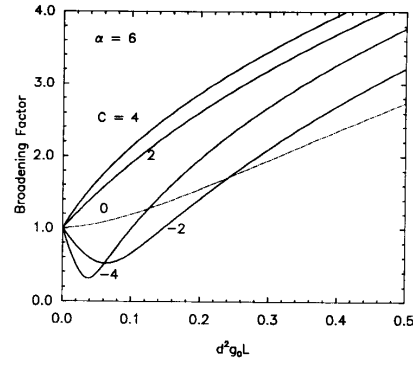$$\cdot \exp\left(- \frac{(1 + iC)\tau^2/2}{1 + d^2 g_0 L(1 - i\alpha)(1 + iC)}\right).$$

(25)

Fig. 1. Broadening factor of the amplified pulses as a function of $d^2 g_0 L$, where d is the dispersion parameter (see text). Different curves correspond to different values of the chirp parameter C associated with the input pulse. Note pulse compression for negative values of C.

The pulse broadening factor is now given by

$$f_B = \left(\frac{[1 + d^2 g_0 L(1 + \alpha C)]^2 + (d^2 g_0 L)^2(\alpha - C)^2}{1 + d^2 g_0 L(1 + \alpha C) - d^2 g_0 L C(\alpha - C)}\right)^{1/2}$$

(26)

and reduces to that given by (20) for $C = 0$. Fig. 1 shows the broadening factor f_B as a function of $d^2 g_0 L$ for several values of C by choosing $\alpha = 6$. The most noteworthy feature of Fig. 1 is that the output pulse can exhibit pulse narrowing for negative values of C. Such a narrowing is similar to the case of optical fibers [13] which can compress a chirped input pulse as long as $\beta_2 C < 0$. Since gain-induced GVD coefficient β_2^{eff} is positive, pulse compression can occur only for negative values of C. Interestingly, pulses emitted by semiconductor lasers are generally chirped such that C is negative. Such pulses can potentially be amplified and compressed simultaneously in semiconductor laser amplifiers if the amplifier gain and the input pulse width are optimized to result in a value of $d^2 g_0 L$ that corresponds to a minimum in Fig. 1. Note that the optimum value of $d^2 g_0 L$ depends on C, and can be quite small (~ 0.01) for large negative values of C. Such small values are possible even for relatively long pulses of width ~ 5 ps. Pulse compression has been observed experimentally [14] and is generally attributed to gain-saturation effects [9], [15]. Our analysis shows that pulse compression can occur even in the linear (unsaturated) regime when chirped picosecond input pulses are amplified.

So far we have assumed that the input pulse is Gaussian. In practice, the pulses emitted by semiconductor lasers may be far from being Gaussian. The effect of pulse shape can be studied by adopting the super-Gaussian model [13] for which the input pulse amplitude is given by

$$U(0, \tau) = \exp[-(1 + iC)\tau^{2m}/2].$$ (27)

The parameter m controls the sharpness of pulse edges and can be related to the rise time [13]. The pulse shape is Gaussian for $m = 1$ but becomes increasingly rectan-
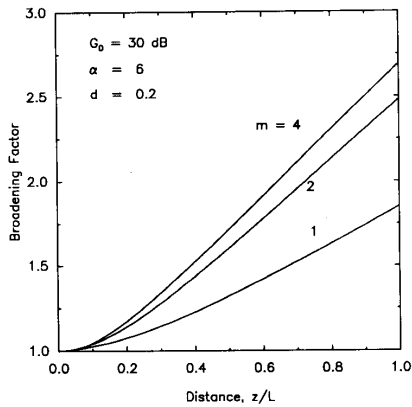
Fig. 2. Broadening of super-Gaussian input pulses along the amplifier length for three values of the parameter m. The input pulse becomes Gaussian for $m = 1$.

gular with sharper leading and trailing edges as m becomes larger. We solved (16) numerically with the initial condition (27) and studied the evolution of the amplified pulse inside the amplifier for several values of m. Since the pulse shape is not Gaussian, the FWHM is not a good measure of the pulsewidth. Fig. 2 shows how the root-mean-square (rms) width of the pulse increases along the amplifier length for $m = 1, 2$, and 4 when the parameters $G_0 = 30$ dB, $\alpha = 6$, $d = 0.2$, and $C = 0$. Super-Gaussian input pulses broaden more than the Gaussian pulse as they propagate inside the amplifier. This can be understood by noting that their spectrum is broader because of sharper pulse edges associated with a super-Gaussian pulse. As a result, the gain-induced GVD is more effective in broadening the input pulse. Similar to the case of Gaussian pulses, super-Gaussian pulses also exhibit an initial compression stage if they are chirped with negative values of the chirp parameter C. The main conclusion of this section is that gain dispersion generally leads to temporal changes because of gain-induced GVD. For suitably chirped input pulses, the amplified pulse may be compressed depending on the amplifier gain and the input pulse width. Both pulse compression and pulse broadening can occur in the linear regime in which the gain remains unsaturated. In the next section, we consider the effect of gain saturation.

IV. SATURATION REGIME

In this section, we consider the general case in which the energy of the optical pulse is comparable to the saturation energy of the amplifier so that the saturation parameter s in (13) is not negligible. When both gain saturation and gain dispersion are included, (13) must be solved numerically. We have used the generalized split-step Fourier method [13] to solve (13). In the following, we discuss our numerical results for an amplifier having a single-pass unsaturated gain of 30 dB. The internal loss is neglected ($\alpha_{int} = 0$) but can easily be included. The line-
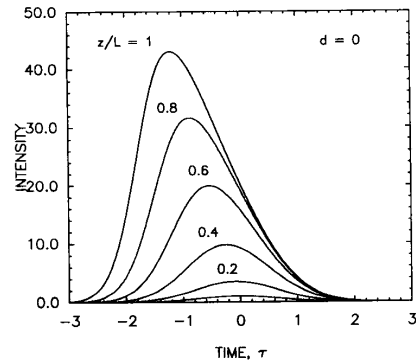

Fig. 3. Evolution of Gaussian input pulse inside the amplifier in the saturation regime for $E_{in}/E_{sat} = 0.1$, $G_0 = 30$ dB, and $\alpha = 6$. Gain dispersion is ignored by setting $d = 0$.
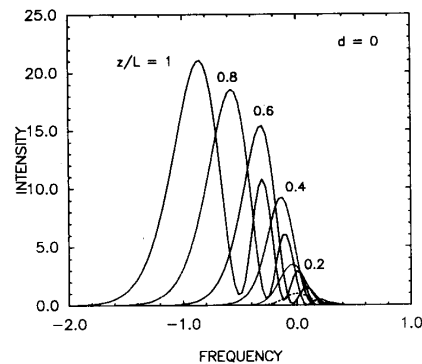

Fig. 4. Pulse spectra corresponding to pulse shapes shown in Fig. 3.

width enhancement factor is taken to be $\alpha = 6$, which is a typical value of 1.55 μm InGaAsP amplifiers. The input energy is taken such that $E_{in}/E_{sat} = 0.1$.

Consider first the case $d = 0$ so that gain dispersion is negligible. Figs. 3 and 4 show the evolution of pulse shape and pulse spectrum for an unchirped Gaussian input pulse. The pulse shape becomes asymmetric and its peak is shifted toward the leading edge. This behavior is easily understood by noting that the leading edge of the pulse saturates the amplifier so that the trailing edge experiences less gain than the leading edge. The pulse spectrum in Fig. 4 exhibits broadening, shifts toward the red side, and develops a multipeak structure as the pulse is amplified [8]. These spectral changes are due to SPM occurring as a result of gain-saturation-induced index changes [9]. Spectral changes occur only for $\alpha \neq 0$ and disappear if α were zero because the refractive index then remains unchanged.

We now consider the effect of gain dispersion on pulse shapes and spectra by taking $d = 0.2$ while all other parameters remain the same. Figs. 5 and 6 show the resulting shapes and spectra, and should be compared to Figs. 3 and 4, respectively, for which $d = 0$. The input Gaussian pulse is distorted considerably and exhibits significant
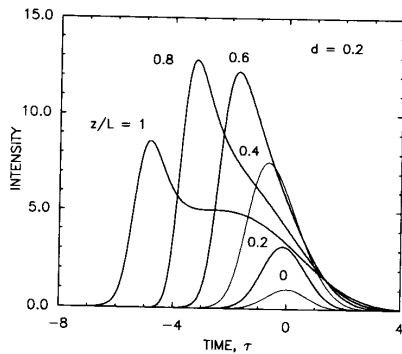
Fig. 5. Pulse shapes under conditions identical to those of Fig. 3, except that gain dispersion is included by choosing $d = 0.2$.
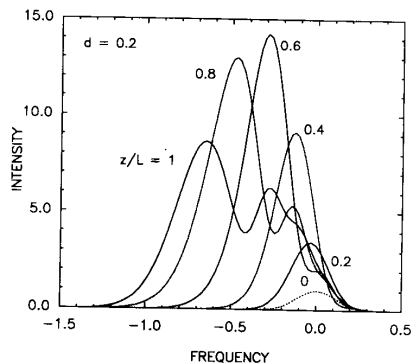

Fig. 7. Pulse shapes at $z/L = 0, 0.5$, and 1 for the case of a super-Gaussian input pulse with $m = 4$. Other parameters are identical to those of Fig. 5.

Fig. 6. Pulse spectra corresponding to pulse shapes shown in Fig. 5.

Fig. 8. Pulse spectra corresponding to pulse shapes shown in Fig. 7.

broadening when $d \neq 0$. In fact, the amplified pulse at $z = L$ in Fig. 3 appears to be narrower than the input pulse in the absence of gain dispersion ($d = 0$) if the comparison is made on the basis of the FWHM. On the other hand, the amplified pulse is much broader than the input pulse when gain dispersion is included. The broadening of input pulses for $d \neq 0$ can be attributed to the gain-induced GVD effects discussed in Section III. The pulse shapes seen in Fig. 5 can also be understood by considering the simultaneous presence of gain dispersion and gain saturation. Gain saturation imposes a chirp on the amplified pulse such that its leading edge is red shifted while the trailing edge is blue shifted [7], [9]. Gain dispersion, on the other hand, induces normal GVD such that red-shifted components travel faster than the blue-shifted components of the pulse. Clearly, the leading edge would arrive earlier while the trailing edge is delayed by the normal GVD. The asymmetric, stretched temporal profiles seen in Fig. 5 result from such an interplay between the effects of gain saturation and gain dispersion.

A comparison of Figs. 4 and 6 shows that the spectrum of the amplified pulse is also considerably affected by gain dispersion. The main change appears to be a broadening of the individual spectral peaks seen in Fig. 4 for $z/L = 1$, such that they overlap and merge into one another. The total spectral extent is slightly reduced because of reduced
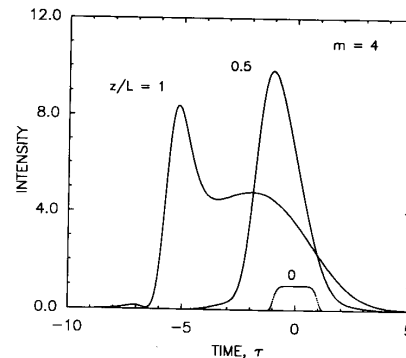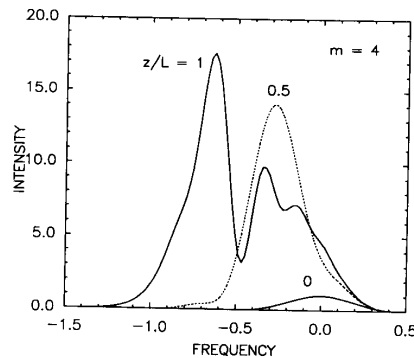
gain in the spectral wings. In particular, the red shift of the dominant peak is less when $d = 0.2$ than when $d = 0$. The interplay between gain-induced GVD and gain-saturation-induced SPM is responsible for these spectral changes.

Temporal and spectral changes seen in Figs. 5 and 6 depend on the input pulse shape quantitatively but not qualitatively. Figs. 7 and 8 show, for example, the pulse shapes and spectra for the case of a super-Gaussian input pulse with $m = 4$ and $C = 0$ in (27). A comparison of Figs. 5 and 7 shows that the amplified pulse at the amplifier output ($z/L = 1$) has qualitatively similar features in both cases even though the input pulse has quite sharp leading and trailing edges in the super-Gaussian case. Pulse spectra are also qualitatively similar in the two cases (compare Figs. 6 and 8), although details are somewhat different.

The results shown in Figs. 3–8 assume that the input pulse is unchirped. Figs. 9 and 10 compare the pulse shapes and spectra at the amplifier output for $C = -2, 0$, and 2, respectively, in order to show the effect of an initial frequency chirp. The amplified pulse is narrowest for $C = -2$ and broadest for $C = 2$. This is not surprising if we refer to Fig. 1, which shows that the pulse is initially compressed for negative values of C in the absence of gain saturation. The initial narrowing stage persists even
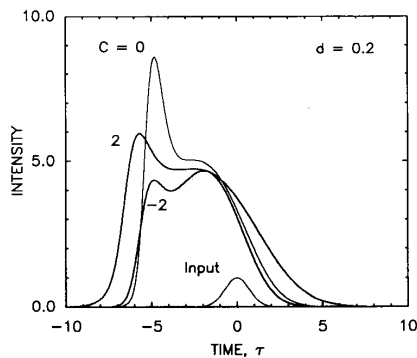
Fig. 9. Effect of initial frequency chirp imposed on the input Gaussian pulse. Amplified pulse shapes at $z = L$ are compared for $C = -2, 0$, and 2.
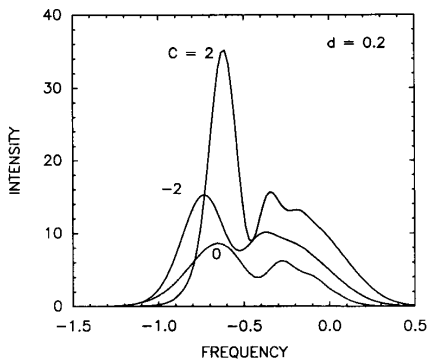

Fig. 10. Pulse spectra corresponding to pulse shapes shown in Fig. 9.

in the presence of gain saturation whose main role is to distort the pulse and broaden the spectrum through SPM.

V. CONCLUDING REMARKS

This paper has discussed the effect of gain dispersion in semiconductor laser amplifiers for input pulses whose width is short enough that the spectral wings experience less amplification than the central spectral components. The effect of gain dispersion becomes increasingly more important as the input pulse gets shorter than 10 ps; the numerical results presented here assumed input pulses of width ~ 1 ps for definiteness. The saturated and unsaturated amplification regimes, corresponding to whether the pulse energy is comparable to or much smaller than the saturation energy of the amplifier, were considered separately. In the unsaturated regime, the gain-induced GVD is found to play an important role. It can lead to pulse narrowing or pulse broadening depending on whether the input pulses were chirped or unchirped. The pulse spectrum remains largely unaffected in the unsaturated regime. In the saturation regime, the pulse spectrum shifts toward the red side and is considerably broadened as a result of gain-saturation-induced nonlinear index changes. When both gain saturation and gain dispersion act together, the shape and the spectrum of the amplified pulse

develop new features which depend on the shape and the chirp of the input pulses.

An interesting issue is related to the existence of solitons or soliton-like pulses in semiconductor laser amplifiers. Since both GVD and SPM are present simultaneously, the two effects may be able to cooperate in such a manner that the amplified pulse evolves toward a soliton whose width, shape, and energy are determined by the amplifier irrespective of the input pulse parameters. Solitons in an amplifying dispersive medium have been discovered in the case of weak saturation [16], [17]. Solitons can also occur in an amplifier with gain dispersion but without gain saturation [18]. Whether solitons can exist when both gain dispersion and gain saturation are present is an issue that deserves further attention.

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Govind P. Agrawal (M'83-SM'86), for a photograph and biography, see p. 343 of the March 1991 issue of this JOURNAL.