Amplification of Ultrashort Solitons in Erbium-Doped Fiber Amplifiers

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Abstract—**Pulse amplification in erbium-doped fiber amplifiers is studied by considering a general model that includes both gain saturation and gain dispersion. The effects of gain dispersion are studied numerically for the case in which a fundamental soliton is launched at the amplifier input. The results show that fiber amplifiers may be useful for simultaneous amplification and compression of weak optical pulses. Under high-gain conditions the input pulse is found to split into a train of amplified subpulses whose width and repetition rate are governed by the gain bandwidth. The numerical results are in qualitative agreement with the recent experiments.**

ERBIUM-DOPED fiber amplifiers (EDFA) are attracting considerable attention because of their potential application in optical communication systems [1]–[3]. The gain bandwidth of such amplifiers is large enough ($\sim$ 10 nm) that gain dispersion is generally negligible for picosecond optical pulses [4]. However, gain dispersion is expected to play an increasingly important role as the pulse becomes shorter than 1 ps [5]–[7]. This letter presents a general formalism for treating pulse amplification in EDFA by including both gain dispersion and gain saturation. The resulting equations are solved numerically to study amplification of fundamental solitons in EDFA. The results show a considerable compression of the fundamental soliton; the width of the amplified pulse is determined by the gain bandwidth [8], [9]. The amplified pulse is also considerably chirped. Under high-gain conditions the input soliton is found to split into a train of ultrashort subpulses whose width is limited by gain dispersion and is generally shorter than 100 fs. The numerical results are in qualitative agreement with the recent experiments [5]–[7].

For an undoped fiber the amplitude of the pulse envelope is obtained by solving a nonlinear Schrödinger equation of the form [10]

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = i\gamma|A|^2A \quad (1)$$

where v_g is the group velocity, β_2 is the group-velocity dispersion (GVD) coefficient, α is the fiber loss, and the nonlinearity parameter $\gamma = \omega_0 n_2/(cA_{\mathrm{eff}})$ at the carrier frequency ω_0. A_{eff} is the effective core area of the fiber. The role of erbium dopants is to provide the gain. If the polarization relaxation time T_2 of the gain medium is short enough to satisfy $\Delta\omega T_2 < 1$, where $\Delta\omega$ is the pulse spectral width, the gain can be included in (1) by replacing α by $\alpha - g$. The effect of gain dispersion is to reduce the gain for spectral components far away from the carrier frequency ω_0. The frequency dependence of the gain can be approximated by $g(\omega) = g_p[1 - (\omega - \omega_0)^2 T_2^2]$ in the vicinity of the gain peak where g_p is the peak gain and ω_0 is assumed to coincide with the gain peak. In the time-domain description $\omega - \omega_0$ is replaced by the operator $i(\partial/\partial t)$, and (1) becomes

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = i\gamma|A|^2A + \frac{1}{2}g_p(t)\left(A + T_2^2\frac{\partial^2 A}{\partial t^2}\right). \quad (2)$$

The time dependence of the peak gain results from gain saturation and is governed by [11]

$$\frac{\partial g_p}{\partial t} = \frac{g_0 - g_p}{T_1} - \frac{g_p|A|^2}{E_s} \quad (3)$$

where T_1 is the population decay time, $E_s = \hbar\omega_0 A_{\mathrm{eff}}/\sigma$ is the saturation energy, and σ is the transition cross section. When the pulse width $T_0 \ll T_1$ ($T_1 \simeq 10$ ms), the T_1 term is negligible in (3) during pulse amplification, and $g_p(t)$ is given by

$$g_p(t) = g_0 \exp\left[-\frac{1}{E_s}\int_{-\infty}^{t} |A|^2\, dt\right] \quad (4)$$

where g_0 is the unsaturated value of the peak gain. For a pulse train g_0 may be different from pulse to pulse if the repetition rate $R \gg 1/T_1$ since there is not enough time for pumping to take place. This case is not considered here.

Equations (2) and (4) govern amplification of ultrashort optical pulses and include the effects of both gain dispersion and gain saturation. They are derived under the assumption that T_2 is smaller than the pulse width T_0. This approximation may not be justified for $T_0 < 100$ fs; coherent effects are expected to play an important role for such short pulses. The extent of gain saturation is governed by E_p/E_s where E_p is the input pulse energy. The saturation energy E_s is $\sim 1\ \mu$J for EDFA.

For the purpose of numerical calculations it is useful to write (2) in the normalized form by using the "soliton units" [10]

$$\tau = (t - z/v_g)/T_0, \quad \xi = z/L_D, \quad U = A/P_0^{1/2} \quad (5)$$

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where $L_D = T_0^2 / |\beta_2|$ is the dispersion length and P_0 is the peak power of the input pulse. Equations (2) and (4) can be combined to yield

$$\frac{\partial U}{\partial \xi} - \frac{i}{2}\frac{\partial^2 U}{\partial \tau^2} + \frac{1}{2}\alpha L_D U = iN^2 |U|^2 U + \frac{1}{2} g_0 L_D \exp\left(-s\int_{-\infty}^{\tau} |U|^2\, dt\right)\left(U + \tau_2^2 \frac{\partial^2 U}{\partial \tau^2}\right) \quad (6)$$

where $\beta_2 < 0$ was assumed (anomalous GVD) and

$$N = (\gamma P_0 L_D)^{1/2}, \quad \tau_2 = T_2 / T_0, \quad s = P_0 T_0 / E_s. \quad (7)$$

Equation (6) is the basic equation of this letter. The parameter N represents the soliton order; $N = 1$ for the amplification of fundamental solitons. The parameter τ_2 accounts for gain dispersion ($\tau_2 < 1$) while the parameter s governs gain saturation ($s \sim E_p / E_s$). For typical fiber parameters $s \sim 10^{-5}$ for $N = 1$, indicating that gain saturation is negligible for amplification of fundamental solitons.

Fig. 1 shows the evolution of a fundamental soliton $[U(0, \tau) = \mathrm{sech}(\tau)]$ for the case $\tau_2 = 0.2$, $s = 0$ and an energy gain of 10 dB per dispersion length. The most noteworthy feature of Fig. 1 is the pulse compression accompanying the amplification process. Such a width reduction can be of considerable practical importance as it can be used to simultaneously amplify and compress weak optical pulses. At $z = L_D$ ($\xi = 1$) the pulse width in Fig. 1 is reduced by about a factor of 10 from its initial value at $z = 0$. As a result, the peak power is enhanced by 18 dB even though the energy gain is only 10 dB. The compressed pulse rides on a pedestal similar to the case of higher-order solitons for fibers without gain. In a sense, the situation is similar in the two cases. The amplification of the input pulse raises its peak power so much that it no longer travels as a fundamental soliton. For $T_2 = 0$ and relatively small values of the amplifier gain the fundamental soliton can maintain itself by simply narrowing its width [8]. Such an adiabatic amplification does not occur in Fig. 1 as the amplifier gain is too high for the soliton to adjust its width adiabatically.

The breakdown of adiabatic amplification is also partly due to the presence of gain dispersion that imposes a frequency chirp $\Delta\upsilon$ on the amplified pulse. Fig. 2 shows the chirp, plotted as $\Delta\upsilon\, T_0$ versus τ, for the pulse shapes of Fig. 1 in the range $\xi = 1$–1.6. At $\xi = 1$ where the pulse is narrowest in Fig. 1, the chirp $\Delta\upsilon$ is quite large. In fact, the pulse spectrum (not shown) exhibits a three-peak structure and is broadened by more than a factor of 6 because of the gain-induced chirp. The chirp is, however, considerably reduced on further propagation of the pulse and exhibits a sign reversal near $\xi = 1.5$ in Fig. 2. Note also that the pulse becomes broader in Fig. 1 for $\xi > 1$. These features can be understood in terms of gain dispersion. The pulse spectrum at $\xi = 1$ is so broad that the EDFA is not able to amplify the spectral wings. As a result, the spectrum narrows, chirp is reduced, and the pulse becomes broader. The process repeats with further propagation.

One may ask what would happen if the EDFA was long

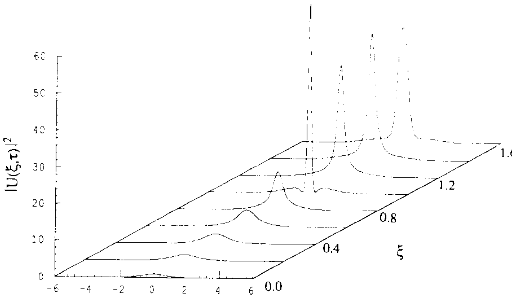

Fig. 1. Evolution of a fundamental soliton inside an EDFA with gain of 10 dB per dispersion length. The other parameters are $s = 0$ and $\tau_2 = 0.2$.

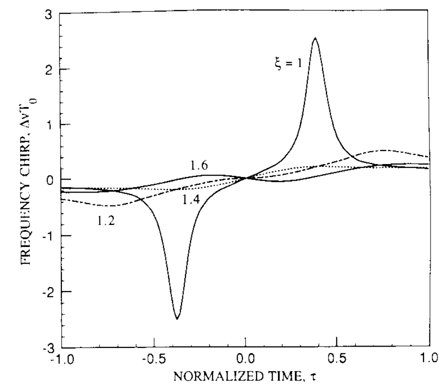

Fig. 2. Frequency chirp imposed on the amplified pulse for ξ in the range 1–1.6. All parameters are the same as in Fig. 1.

enough to permit soliton amplification over many dispersion lengths. The question can be answered by investigating the soliton-like solutions of (6). This equation is known to have a solitary-wave solution when $s = 0$ [9]. The solution is in the form of a chirped soliton whose intensity profile is given by a hyperbolic secant but the width is governed by the gain-dispersion parameter T_2 rather than the width of the input soliton. For the parameter values used in Fig. 1 the asymptotic width is approximately $T_2/\sqrt{3}$. Thus, the input soliton of Fig. 1 is expected to evolve asymptotically toward a chirped soliton whose width is reduced by a factor of $\sqrt{3}/\tau_2$ or about 8.7. This is indeed what happens for $\xi > 1$ in Fig. 1; the pulse broadens in an attempt to evolve toward the chirped solitary-wave solution of (6).

The qualitative behavior of soliton amplification can be quite different for the case of high amplifier gain. Fig. 3 shows the pulse shapes at $\xi = 1$ and $\xi = 2$ for parameter values identical to those of Fig. 1 except that the amplifier gain is now 20 dB per dispersion length. The pulse shape at $\xi = 1$ is similar to the 10-dB-gain case shown in Fig. 1. However, pulse evolution beyond $\xi = 1$ exhibits a qualitatively different behavior. In particular, the amplified pulse splits into multiple soliton-like pulses of about the same width. This is seen clearly in Fig. 2 for $\xi = 2$. A simple interpretation is that each pulse is a chirped soliton. The chirped soliton of (6) correspond to a definite value of peak power and energy [9]. When the peak power exceeds this value, the pulse splits into multiple pulses which then evolve

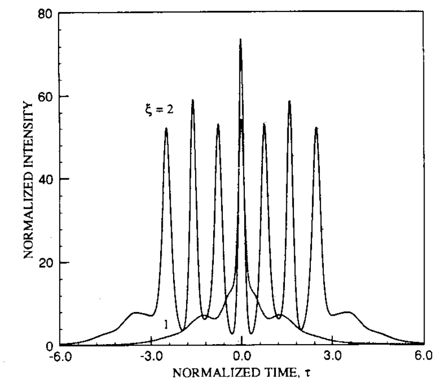

Fig. 3. Pulse shapes at $\xi = 1$ and $\xi = 2$ for an EDFA with gain 20 dB per dispersion length. The other parameters are identical to those of Fig. 1.

to become chirped solitons. The pulse spectra appear to support this interpretation. From a practical standpoint, the input pulse is converted into a train of ultrashort pulses whose width and repetition rate are governed by the amplifier parameters. Since the width is a fraction of T_2, it is generally shorter than 100 fs for EDFA.

Some of the features of pulse amplification discussed above have been observed in the recent experiments [5]-[7]. In one experiment [6] pulse compression was observed in an EDFA of 55-cm length. In another experiment [7] compression of a 250-fs input pulse by a factor of up to 4 was observed. In an interesting experiment [6] a 4-m-long fiber with the net gain of 21.5 dB showed the splitting of the input pulse into at least five separate pulses. From the numerical results presented here it appears that the effects of gain dispersion (finite T_2) have been observed in this experiment.

In conclusion, amplification of ultrashort solitons in EDFA is discussed by considering a general model that is capable of including both gain saturation and gain dispersion. The effects of gain dispersion are studied numerically for the case in which a fundamental soliton is launched at the amplifier input. The results show that EDFA's may be useful for simultaneous amplification and compression of weak optical pulses. Under high-gain conditions the input pulse is split into a train of chirped solitons whose width is governed by T_2 and is generally shorter than 100 fs. The numerical results are in qualitative agreement with the recent experiments. The model is currently being extended to include the effects of intrapulse stimulated Raman scattering and the resulting soliton self-frequency shift.

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