Spectrum of partially coherent light: transition from near to far zone

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Received 26 January 1990

The evolution of correlation-induced spectral shifts with propagation in free space is investigated in the paraxial regime with particular emphasis on the transition from near to far zone. The dependence of spectral shifts on the state of coherence, the angle of observation, and the source linewidth is studied for partially coherent light generated by planar secondary gaussian Schell-model sources.

1. Introduction

It has recently been shown that the spectrum of light produced by partially coherent sources generally changes on propagation, an effect sometimes called the *Wolf effect*. The extensive work of Wolf and coworkers [1–5] has demonstrated that the spectrum of partially coherent light can shift toward shorter or longer wavelengths, depending on the coherence properties of the source, even when the light propagates in free space. Most of the previous work has focused on spectral changes occurring in the far zone of partially coherent sources [5]. Although correlation-induced spectral changes are expected to evolve continuously from the near zone to the far zone, it is now clear how far away the light can propagate from the source plane before such changes become significant.

In this paper we address the issue of the evolution of spectral shifts from the near to the far zone by deriving an expression for the field spectrum that is valid, within the paraxial approximation, for an arbitrary propagation distance in free space. We use this expression to obtain the spectral shift for optical fields generated by a gaussian Schell-model source, and to study how the shift changes during transition from the near to the far zone. The development of spectral shifts depends on the state of coherence of the source. For relatively incoherent sources and small observation angles, the spectrum is found to shift toward the blue side, and the maximum blue shift is attained after the light has propagated a distance of only a few wavelengths ($z \leqslant 10\lambda$). By contrast, for a relatively coherent source the spectrum, viewed at a fixed angle, first shifts toward blue and then toward red as the propagation distance increases. We discuss the development of the spectrum during transition from the near to the far zone in detail, with particular emphasis on its dependence on the spectral line width of the source spectrum and the direction of observation.

2. Near-zone spectrum for gaussian Schell-model sources

Consider a secondary source located in the plane $z=0$ and radiating into the half-space $z>0$. If we denote the cross-spectral density of the source by $W(\boldsymbol{R}_1, \boldsymbol{R}_2; \omega)$, then the field spectrum [#1] at an arbitrary point $\boldsymbol{r}$ is given by [6]

[#1] Here the field spectrum $S(\boldsymbol{r}; \omega)$ is taken to be the diagonal component $W(\boldsymbol{r}, \boldsymbol{r}; \omega)$ of the cross-spectral density. Strictly speaking, the spectral intensity should be identified with the magnitude of the flux vector associated with the optical field. However, in the paraxial approximation these two quantities are proportional to each other.

$$S(\boldsymbol{r};\omega)=\iint K^*(\boldsymbol{r},\boldsymbol{R}_1;\omega)\,K(\boldsymbol{r},\boldsymbol{R}_2;\omega)\times W(\boldsymbol{R}_1,\boldsymbol{R}_2;\omega)\,\mathrm{d}^2R_1\,\mathrm{d}^2R_2\,, \tag{1}$$

where the integration is to be performed twice over the source plane. In the angular spectrum representation, the kernel K in eq. (1) for propagation in free space is given by [7]

$$K(\boldsymbol{r},\boldsymbol{R};\omega)=\frac{k^2}{4\pi^2}\iint_{-\infty}^{\infty}\exp\{\mathrm{i}k[p(x-X)+q(y-Y)+mz]\}\,\mathrm{d}p\,\mathrm{d}q\,, \tag{2}$$

where $k=\omega/c$ (c is the speed of light in vacuum) is the wavenumber associated with frequency ω, and

$$\begin{aligned} m&=(1-p^2-q^2)^{1/2}\,, &&\text{when } p^2+q^2\leqslant 1\,,\\ &=\mathrm{i}(p^2+q^2-1)^{1/2}\,, &&\text{when } p^2+q^2>1\,. \end{aligned} \tag{3}$$

The integration in eq. (2) is taken formally over all (p, q) space. In the paraxial approximation the amplitudes of the angular spectrum of the field are negligible unless $p^2+q^2\ll 1$, and m can be approximated by

$$m\approx 1-(p^2+q^2)/2\,. \tag{4}$$

By substituting eq. (4) in eq. (2) and performing the integration, one can show that the propagation kernel takes the form

$$K(\boldsymbol{r},\boldsymbol{R};\omega)=\frac{k\exp(\mathrm{i}kz)}{2\pi\mathrm{i}z}\times\exp\{(\mathrm{i}k/2z)[(x-X)^2+(y-Y)^2]\}\,. \tag{5}$$

We note that this kernel, when substituted into eq. (1), is equivalent to the use of the Fresnel approximation for the propagation of the cross-spectral density.

We now consider the cross-spectral density of a gaussian Schell-model source [8], viz.,

$$W(\boldsymbol{R}_1,\boldsymbol{R}_2;\omega)=S^{(0)}(\omega)\times\exp\left(-\frac{\boldsymbol{R}_1^2+\boldsymbol{R}_2^2}{4\sigma_I^2}-\frac{(\boldsymbol{R}_2-\boldsymbol{R}_1)^2}{2\sigma_g^2}\right). \tag{6}$$

Here σ_I is the rms intensity width and σ_g is the rms spatial correlation distance. Both σ_I and σ_g may depend on ω in general. The source spectrum $S^{(0)}(\omega)$ is assumed to be the same at all points in the source domain. On substituting eqs. (5) and (6) into eq. (1) and performing the four dimensional integration we obtain the following expression for the field spectrum:

$$S(\boldsymbol{r};\omega)=S^{(0)}(\omega)\,M(\boldsymbol{r};\omega)\,, \tag{7}$$

where $M(\boldsymbol{r};\omega)$ is the "spectral modifier" given by the expression

$$M(\boldsymbol{r};\omega)=\frac{1}{1+(z/z_\mathrm{d})^2}\times\exp\left(-\frac{x^2+y^2}{2\sigma_I^2}\frac{1}{1+(z/z_\mathrm{d})^2}\right), \tag{8}$$

with

$$z_\mathrm{d}=\frac{2k\sigma_I^2}{(1+4\sigma_I^2/\sigma_g^2)^{1/2}}\,. \tag{9}$$

The parameter z_d is analogous to the so-called diffraction length or the Rayleigh range [9] encountered in propagation of coherent gaussian beams and reduces to it in the coherent limit ($\sigma_g\gg\sigma_I$). The spectral changes occurring on propagation are due to the frequency dependence of z_d. The state of spatial coherence of the source affects the value of z_d. In the incoherent limit $\sigma_g\ll\sigma_I$, and consequently $z_\mathrm{d}=k\sigma_I\sigma_g$. For our later discussion we define the parameter

$$z_0=z_\mathrm{d}\,|_{k=k_0}=\frac{2k_0\sigma_I^2}{(1+4\sigma_I^2/\sigma_g^2)^{1/2}}\,, \tag{10}$$

where $k_0=\omega_0/c=2\pi\nu_0/c$, and ν_0 is the frequency at which the source spectrum $S^{(0)}(\omega)$, assumed to consist of a single spectral line, peaks. We shall refer to z_0 as the effective diffraction length for partially coherent gaussian beams.

The spectrum of the field $S(\boldsymbol{r};\omega)$ given by eq. (7) is valid for any propagation distance within the paraxial regime. In particular, it follows from eq. (8) that in the limit $z\to 0$ we recover, as expected, the source spectrum, i.e.,

$$S(x,y,0;\omega)=S^{(0)}(\omega)\exp\left(-\frac{x^2+y^2}{2\sigma_I^2}\right). \tag{11}$$

Similarly, for propagation to the far zone ($z\gg z_\mathrm{d}$) the spectral modifier (8) becomes

$$M(r;\omega)=\left(\frac{z_d}{z}\right)^2 \exp\left[-\frac{(x^2+y^2)}{2\sigma_I^2}\left(\frac{z_d}{z}\right)^2\right], \quad (12)$$

and, apart from notational differences, is in agreement with a previously derived result [10].

Another interesting limit of eqs. (7) and (8) concerns quasi-homogeneous sources [11]. These sources are essentially globally incoherent, i.e. they satisfy the relation $\sigma_g \ll \sigma_I$. Wolf has shown [1] that when sources of this type satisfy a certain scaling law, the spectrum of the field in the far zone is independent of the direction of observation and is proportional to the source spectrum. In order that a gaussian Schell-model source satisfies the scaling law one must have

$$\sigma_g=\sigma_g(\omega)\equiv\alpha/k\,, \quad (13)$$

where α is an arbitrary positive constant. Since $z_d=k\sigma_I\sigma_g$ for quasi-homogeneous sources, it follows that for sources that satisfy the scaling law z_d is frequency independent when σ_I and σ_g are also independent of frequency. Moreover, since the spectral modifier becomes frequency independent, the spectrum of the field remains invariant for any propagation distance. Although the scaling law was derived originally [1] in connection with the spectrum in the far-zone, our analysis shows that for quasi-homogeneous gaussian Schell-model sources it is applicable in the paraxial approximation even in the near zone [#2]. In fact, one can show that for all quasi-homogeneous sources obeying the scaling law the spectrum of the field remains invariant for all propagation distances within the region of validity of the paraxial approximation.

3. Evolution of spectral shifts on propagation from near to far zone

In this section we examine the evolution of spectral shifts as a function of the propagation distance. We also investigate the effects of the spectral linewidth and the observation angle on the magnitude of the shifts. For definiteness, we assume that the spectrum of the planar secondary source consists of a single line of a lorentzian profile centered at $\lambda_0=600$ nm, and its full width at half maximum (fwhm) varies in the range $\Delta\nu/\nu_0=0$–0.2. We also assume that the width of the intensity profile is fixed at $\sigma_I=20/k_0$. We define the spectral shift $\delta\nu$ as the frequency difference $\nu_s-\nu_0$, where ν_s is the frequency at which $S(r;\omega)$ peaks and ν_0 is the frequency at which $S^{(0)}(\omega)$ peaks ($\nu_0=c/\lambda_0=5\times10^{14}$ Hz). We express our results as a relative frequency shift $\delta\nu/\nu_0$ to emphasize that similar behavior is expected for sources emitting light at other wavelengths. We note that the line shape also generally changes on propagation; this aspect is not considered in this paper.

In figs. 1a–1d we show the calculated spectral shifts for observation in a direction making an angle of 10° from the z-axis, and for propagation distance k_0z in the range from 0 to 1000. Fig. 1a shows the spectral shifts for light produced by a source that is relatively incoherent ($\sigma_g/\sigma_I=0.05$). The spectral shifts develop very rapidly with increasing propagation distance; most of the shift is already present at a distance $z=100/k_0$. The magnitude of the spectral shifts, for sources of this state of coherence, is positive (blue shift) and increases with increasing linewidth of the source spectrum.

Fig. 1b shows the spectral shifts for light from a source whose spatial correlation distance is larger than that used in fig. 1a by a actor of 5 ($\sigma_g/\sigma_I=0.25$). Although the spectral shifts are again toward the blue side and increase with increasing linewidth of the source spectrum, the magnitude of the shifts, at any propagation distance, is reduced compared with those shown in fig. 1. Note that the transition to the far zone now occurs for larger values of k_0z.

Both of the sources considered in figs. 1a and 1b are relatively incoherent. An interesting phenomenon occurs when the correlation distance σ_g is comparable to the intensity width σ_I. Fig. 1c shows the development of spectral shifts as a function of the propagation distance for a source for which $\sigma_g/\sigma_I=0.5$. The spectral lines are blue-shifted for short propagation distances but they exhibit red shifts for distances $z>z_m$, where z_m is the value of z at which the shift is zero. The exact value of z_m at which the changeover from blue to red shift occurs is difficult to obtain analytically; typically $z_m\sim z_0$. The magnitude of the shift is larger for sources whose spectral line is wider except at the propagation distance z_m where the shift vanishes identically. In fig. 1d we

[#2] A similar result was derived in sec. 4 of ref. [5].

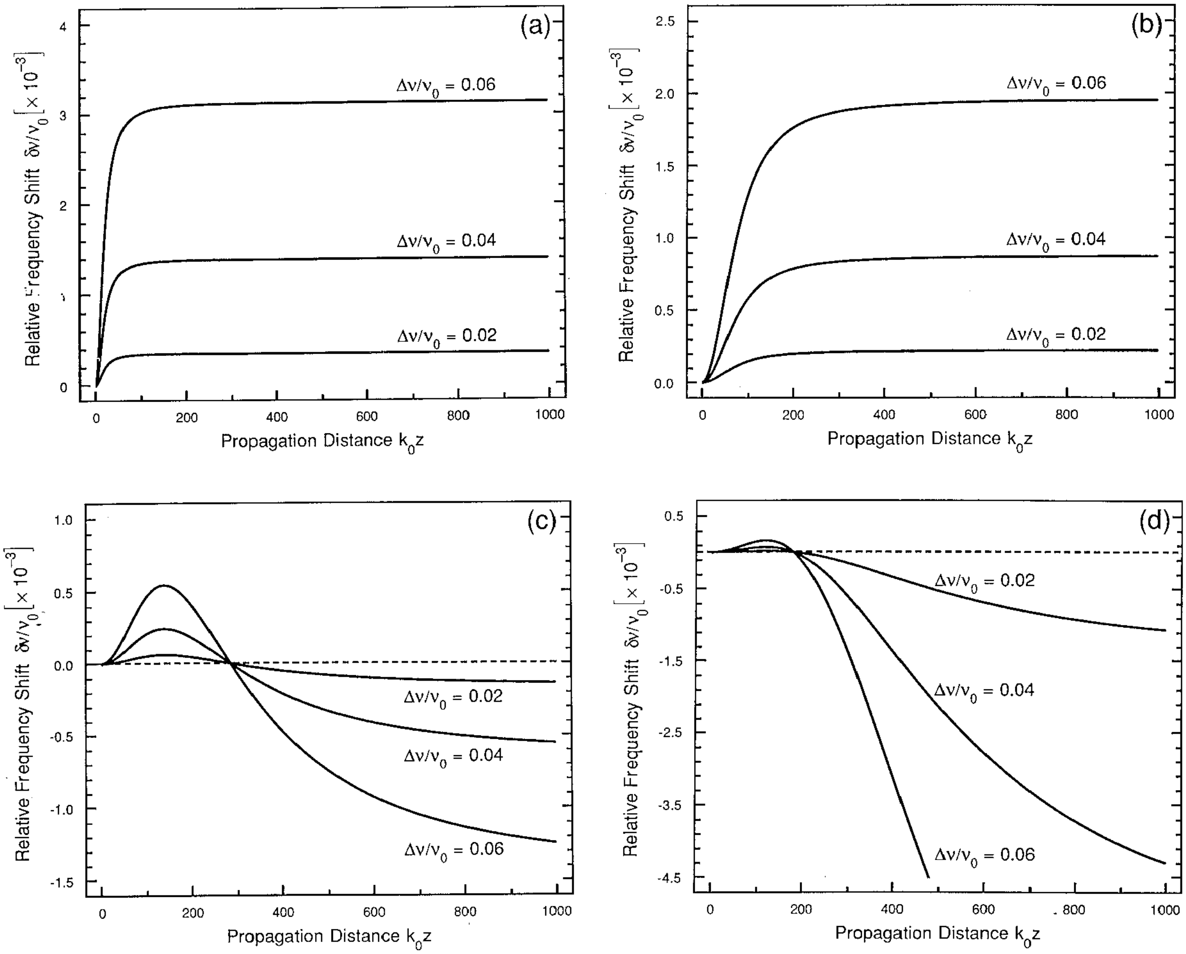

Fig. 1. Relative frequency shifts $\delta\nu/\nu_0$ versus propagation distance for sources having different states of coherence: (a) $\sigma_g/\sigma_I=0.05$, (b) $\sigma_g/\sigma_I=0.25$, (c) $\sigma_g/\sigma_I=0.5$, and (d) $\sigma_g/\sigma_I=1$. In each case three different curves correspond to the different spectral widths of the source. The parameter $k_0\sigma_I=20$.

show the development of spectral shifts for sources for which $\sigma_g/\sigma_I=1$. The increase in the correlation length leads to a larger red shift for $z>z_m$ and a smaller blue shift for $z<z_m$. The magnitude of the shift is again larger for sources having a wider spectral line.

From a practical standpoint, the quantity of interest is the maximum far-zone spectral shift and its dependence on the source parameters such as the spectral width and the state of coherence of the source. The dependence of the spectral shift on the linewidth of the source is shown in fig. 2 for several states of coherence, specified by the ratio σ_g/σ_I for a fixed distance $k_0z=1000$. The other parameters are identical to those of fig. 1. The spectral shifts for quasi-homogeneous sources ($\sigma_g/\sigma_I\ll 1$) are seen to be toward blue while for more coherent sources the shifts occur toward the red. In both cases the spectral shift increases in magnitude with increasing source linewidth.

The calculations shown in figs. 1 and 2 were performed for a fixed angle of observation. The dependence of the far-zone spectral shifts on the direction of observation is shown in fig. 3 as a function of the ratio σ_g/σ_I. When the observation point is on the z-axis, the only modification of the spectrum is due to

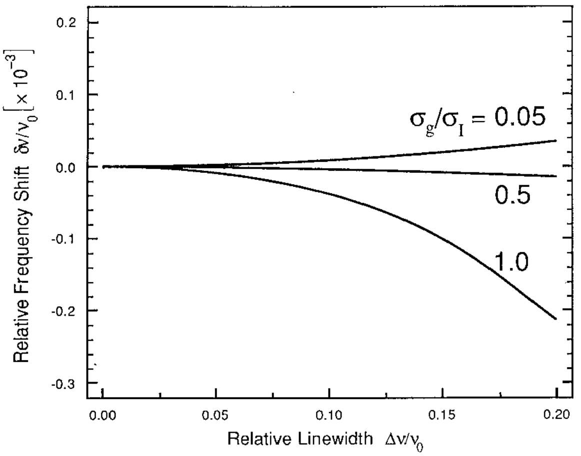

Fig. 2. Relative frequency shifts $\delta\nu/\nu_0$ versus source linewidth $\Delta\nu/\nu_0$ for sources characterized by $k_0\sigma_I=20$ and $\sigma_g/\sigma_I=0.05$ (a), $\sigma_g/\sigma_I=0.5$ (b), $\sigma_g/\sigma_I=1.0$ (c), and $\sigma_g/\sigma_I=1.2$ (d).

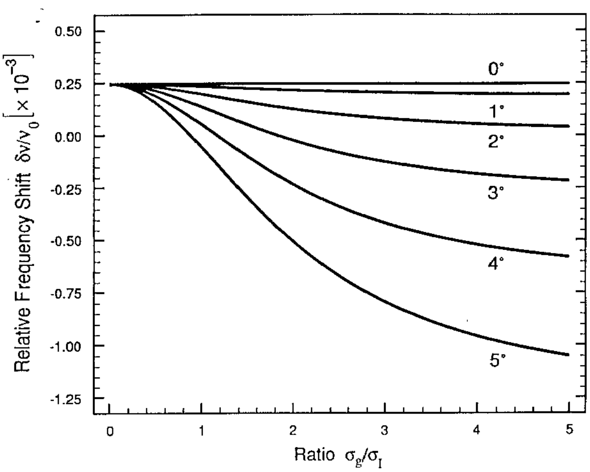

Fig. 3. Relative frequency shifts $\delta\nu/\nu_0$ as a function of source coherence parameter (σ_g/σ_I) for several observation angles in the paraxial region.

the factor $(z_d/z)^2 \propto k^2$ in eq. (8). This is evident in the dashed curve showing a slight blue shift that is independent of the state of coherence. For observation at small angles we see a blue shift for small values of σ_g/σ_I, which develops into a red shift with increasing spatial correlation distance.

4. Discussion

In this letter we examined, within the accuracy of the paraxial approximation, the behavior of correlation-induced spectral shifts as a function of the propagation distance from the source plane. Our results show that when the source is relatively incoherent, spectral blue shifts develop quickly over a very short propagation distance. By contrast, relatively coherent sources exhibit an initial small blue shift that changes to red shift for $z>z_0$. The parameter z_0, referred to here as the effective diffraction length, plays a critical role in determining the transition from near to far zone. The transition takes place for $z\sim z_0$. Since $z_0 \approx k_0\sigma_I\sigma_g$ for quasi-homogeneous sources for which $\sigma_g \ll \sigma_I$, the transition is complete for relatively short propagation distances ($z \leqslant 10\lambda_0$). By contrast, $z_0 = 2k_0\sigma_I^2$ for a coherent source and can be greater than $10^3\lambda_0$ depending on the intensity width σ_I. Although our analysis is confined to the paraxial regime, we showed that, except for very small observation angles, the far-zone spectrum shifts toward the red and the shift increases with increasing observation angles for almost all states of coherence of the source. At any observation angle θ, excluding $\theta=0$ (the z-axis), the magnitude of the spectral shifts was found to increase with increasing source linewidth, irrespective of the state of coherence of the source. Our analysis also demonstrated that, within the paraxial approximation, quasi-homogeneous gaussian Schell-model sources that obey the scaling law produce a field spectrum that is independent of the propagation distance and the observation angle.

Acknowledgements

The authors thank Professor E. Wolf for helpful discussions concerning this manuscript. This research was supported by the U.S. Army Research Office and by the New York State Science and Technology foundation.

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