Effect of Four-Wave Mixing on Multichannel Amplification in Semiconductor Laser Amplifiers

GOVIND P. AGRAWAL, SENIOR MEMBER, IEEE, AND ISAM M. I. HABBAB, MEMBER, IEEE

Abstract—**A general theory is developed to describe multichannel amplification in semiconductor laser amplifiers. It includes the effects of both gain saturation and carrier-density modulation. The carrier-density modulation, occurring as a result of beating among the neighboring channels, is responsible for nondegenerate four-wave mixing. This nonlinear phenomenon not only unequalizes the channel gains but also generates new sidebands on both sides of the boundary channels. We illustrate the effects of four-wave mixing on multichannel amplification by considering the 30 channel case in detail. The theory is useful for quantifying the amplifier-induced crosstalk and identifying the operating conditions for reducing such crosstalk in multichannel coherent communication systems.**

I. Introduction

Semiconductor laser amplifiers have attracted considerable attention recently because of their potential application in fiber-optic communication systems [1]. A major motivation behind their development is the possibility of simultaneous amplification of a large number of closely spaced optical channels. This can be realized only if the interchannel crosstalk is not a limiting factor. Two physical mechanisms responsible for such a crosstalk are gain saturation [2]–[5] and carrier-density modulation [6]–[15]. Gain saturation does not depend on channel spacing and can be avoided by operating the amplifier below the saturation level. By contrast, carrier-density modulation is expected to set a fundamental limit on channel spacing even in the below-saturation mode of operation [7].

The nonlinear phenomenon limiting the amplifier performance is four-wave mixing (FWM) occurring as a result of carrier-density modulation that creates the dynamic gain and index gratings [6], [12]. Although FWM in semiconductor laser amplifiers has been studied [6]–[15], most of the theoretical models use various approximations that limit their range of validity. In this paper we develop a general theory of FWM to describe multichannel amplification by including the effects of both gain saturation and carrier-density modulation. FWM not only affects the channel gains but also generates new sidebands beyond the boundary channels. We illustrate these effects by considering the two-channel case. The theory is then applied to study interchannel crosstalk and intermodulation distortion occurring when 30 channels are amplified simultaneously. The theoretical model used here is valid in the large-signal regime and is solved exactly except for the approximations inherent in the rate-equation approach.

Manuscript received February 21, 1989; revised June 23, 1989.

G. P. Agrawal is with the Institute of Optics, University of Rochester, Rochester, NY 14627.

I. M. I. Habbab is with AT&T Bell Laboratories, Crawford Hill Laboratory, Holmdel, NJ 07733.

IEEE Log Number 8933362.

II. Theory

We consider a traveling-wave amplifier which is used to amplify M equispaced channels at the frequencies ω_m ($m = 1$ to M). All waves are assumed to be monochromatic and linearly polarized along the same axis. The total electric field is then of the form

$$E(x, y, z, t) = U(x, y) \sum_{j=1}^{M} E_j(z) \exp(-i\omega_j t) \quad (1)$$

where $U(x, y)$ is the transverse distribution of the single mode supported by the amplifier waveguide. The evolution of $E(x, y, z, t)$ along the amplifier is governed by the wave equation

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2} \quad (2)$$

where n is the refractive index, c is the vacuum velocity of light, and ϵ_0 is the vacuum permittivity. The induced polarization is calculated by using [6]

$$P = \epsilon_0 \chi(N) E \quad (3)$$

where the susceptibility

$$\chi(N) = -\frac{nc}{\omega_0}(\beta + i) g(N). \quad (4)$$

The gain $g(N)$ depends on the carrier density N inside the amplifier's active region which in turn depends on the pumping current. To a good approximation, $g(N)$ varies linearly with N, i.e., $g(N) = a(N - N_0)$, where a is the gain coefficient and N_0 is the carrier density required for transparency [16]. The carrier density N at a pumping current I is obtained by solving the rate equation

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_s} - \frac{g(N)}{\hbar\omega_0}\langle |E|^2 \rangle \quad (5)$$

0018-9197/90/0300-0501$01.00

where q is the electronic charge, V is the active volume, and τ_s is the spontaneous carrier lifetime. The angle brackets denote averaging over the waveguide cross section. The axial variation of N along the propagation direction z is retained in view of the growth of the channel intensities. The parameter β in (4) accounts for the carrier-induced index change that accompanies the gain in semiconductor lasers and amplifiers. It is often referred to as the linewidth enhancement factor and has typical values in the range 3–6 depending on the operating wavelength [16].

The stimulated-recombination term in (5) is responsible for carrier-density modulation at the beat frequencies Ω, $2\Omega, \cdots, (M-1)\Omega$, where Ω is the channel spacing. An approximate solution of (5) is of the form

$$N(z, t) = \overline{N}(z) + \sum_{m=1}^{M-1} [\Delta N_m(z) \exp(-im\Omega t) + \text{c.c.}] \tag{6}$$

where $\Omega = |\omega_{j+1} - \omega_j|$,

$$\overline{N}(z) = N_0 + \frac{N_0(I/I_0 - 1)}{1 + \sum_j |E_j(z)|^2/P_s} \tag{7}$$

with $I_0 = qVN_0/\tau_s$ and

$$\Delta N_m(z) = -\frac{C(\overline{N}(z) - N_0) \sum_j E_j(z) E^*_{j-m}(z)}{1 - im\Omega\tau_s + \sum_j |E_j(z)|^2/P_s}. \tag{8}$$

The saturation power

$$P_s = \hbar\omega_0 wd/(\Gamma a \tau_s) \tag{9}$$

where w is the width and d is the thickness of the active region. The confinement factor Γ and the overlap factor C result from the non-plane wave nature of the waveguide mode [12]. Typically, $\Gamma = 0.3 - 0.4$ and $C = 0.5 - 1$ depending on the waveguide dimensions. The saturation power P_s is assumed to be the same for all channels by using an average photon energy $\hbar\omega_0$. The modulation component ΔN_m in (6) couples the three waves at the frequencies ω_j, ω_{j-m}, and ω_{j+m} through the FWM process. The power dependence of the denominator in (7) and (8) is responsible for gain saturation.

To solve the wave equation (2), we follow the standard procedure [12]. The polarization P is expanded in a manner similar to (1) by writing

$$P(x, y, z, t) = U(x, y) \sum_{j=1}^{M} p_j(z) \exp(-i\omega_j t). \tag{10}$$

If we substitute (1) and (10) in (2), assume $E_j = A_j \sqrt{P_s} \exp(ik_j z)$, neglect d^2A_j/dz^2, and integrate over x and y, we obtain

$$\frac{dA_j}{dz} + \alpha_j A_j = \frac{i\omega_j \Gamma}{nc\epsilon_0 \sqrt{P_s}} p_j \exp(-ik_j z) \tag{11}$$

where $k_j = n\omega_j/c$. The frequency dependence of n has been ignored by assuming that the channel separation is much smaller than the optical frequencies. This is the case in practice ($\Omega/\omega_j < 10^{-5}$). It amounts to assuming that the FWM process is nearly phase matched over the amplifier length. The loss term has been added phenomenologically to account for the waveguide losses. The polarization component p_j is obtained by using (6) in (3) and expressing the result in the form of (10). The final result is

$$\begin{aligned}\frac{dA_j}{dz} &+ \alpha_j A_j \\ &= \frac{g_0}{2}\frac{1 - i\beta}{1 + P_T}\left[A_j - C\sum_{m=1}^{M-1}\frac{A_{j-m}\left(\sum_k A^*_{k-m}A_k\right)}{1 + P_T - im\Omega\tau_s}\right. \\ &\qquad \left. - C\sum_{m=1}^{M-1}\frac{A_{j+m}\left(\sum_k A^*_k A_{k-m}\right)}{1 + P_T + im\Omega\tau_s}\right]\end{aligned} \tag{12}$$

where $g_0 = \Gamma a N_0(I/I_0 - 1)$ is the small-signal gain and P_T is the total power in all channels (normalized to the saturation power):

$$P_T = \sum_{j=1}^{M} P_j = \sum_{j=1}^{M} |A_j|^2 = \frac{1}{P_s}\sum_{j=1}^{M} |E_j|^2. \tag{13}$$

Equation (12) is our main result. It describes how the complex amplitude of each channel evolves along the amplifier by including the effects of both gain saturation and carrier-density modulation. The latter contribution becomes negligible for $\Omega\tau_s >> 1$ since carriers cannot respond fast enough to changes in the amplitudes if Ω exceeds much more than τ_s^{-1}. In general, the indexes j and k in (12) take integer values which extend beyond the number of channels M since FWM creates new frequency components on both sides of the boundary channels. In practice, it is often sufficient to consider only $M - 1$ additional channels on each side. One then solves (12) with $j = -(M - 2)$ to $2M - 1$ by setting $A_j(0) = 0$ for values of j outside the range $1, 2, \cdots, M$. Since A_j is complex, one needs to solve numerically a set of $6M-4$ first-order nonlinear differential equations to solve the M-channel-amplification problem.

III. Four-Wave Mixing and Gain Asymmetry

Carrier-density modulation and the associated nonlinear phenomenon of FWM affect the performance of amplifiers in two ways. First, the amplifier gain becomes different for different channels. As a result, the output channel powers become unequal even if all channels have the same power at the amplifier input. Second, some energy is transferred to new sidebands located on each side of the boundary channels as a result of FWM, making the amplifier less efficient. We illustrate the two effects by solving (12) for the specific case of two-channel amplification. The amplifier is characterized by the unsaturated single-channel gain defined as

$$G_A = \exp(g_0 L). \tag{14}$$

We choose $G_A = 25$ dB and $\beta = 6$ as typical values for semiconductor laser amplifiers. The carrier lifetime $\tau_s = 0.1$–0.5 ns depending on the residual facet reflectivities and the pumping conditions. However, it is not necessary to specify τ_s if we present our results for a normalized channel spacing $D\tau_s$, where $D = \Omega/2\pi$, and normalize channel powers to the saturation power P_s defined by (9); $P_s \simeq 1$ mW under typical operating conditions. The waveguide loss is neglected.

Fig. 1 shows the channel gains G_1 and G_2 defined as

$$G_j = \frac{|A_j(L)|^2}{|A_j(0)|^2} \tag{15}$$

for $j = 1$ and 2. For simplicity, we assume that the incident channel power $P_{\text{in}} = |A_j(0)|^2$ is the same for both channels. Two cases of $P_{\text{in}} = -30$ and -25 dB are shown in Fig. 1. In the former case the gain-saturation effects are much smaller compared with the latter case. Horizontal dashed lines show the saturated channel gains expected in the absence of FWM. The effect of FWM is to increase the gain for the low-frequency channel and to decrease it for the high-frequency channel [7], [15]. The gain difference between the two channels can be 3 dB or more for $D\tau_s \leq 1$. It can be reduced by operating the amplifier well below the saturation level and by choosing the channel spacing in excess of τ_s^{-1}. Since τ_s is typically 0.2–0.3 ns, the channel spacing for $P_{\text{in}} = -30$ dB must exceed 3–5 GHz to maintain the channel gains within 1 dB of each other.

It is seen in Fig. 1 that both G_1 and G_2 decrease significantly for $D\tau_s \ll 1$. This decrease is due to the transfer of power from each channel to new frequency components at $\omega_1 - \Omega$ and $\omega_2 + \Omega$ ($\omega_1 < \omega_2$ is assumed). These sidebands are generated as a result of FWM. More specifically, when the channel 1 acts as a pump, two photons of the frequency ω_1 are annihilated to create two photons at the frequency $\omega_2 = \omega_1 + \Omega$ and $\omega_0 = \omega_1 - \Omega$. Similarly, a part of the energy of channel 2 is transferred to frequency components at $\omega_1 = \omega_2 - \Omega$ and $\omega_3 = \omega_2 + \Omega$. The frequency components at ω_0 and ω_3 have no power present initially at the ampilfier input. However, FWM can generate a significant amount of power at these frequencies. Fig. 2 shows the powers $P_0 = |A_0(L)|^2$ and $P_3 = |A_3(L)|^2$ (normalized to P_s) at the two sidebands generated as a result of FWM. In general, the low-frequency sideband at ω_0 has more power than the high-frequency sideband at ω_3, although the difference is within 1–2 dB. Both P_0 and P_3 decrease rapidly with an increase in the channel spacing. Note however that the sideband power

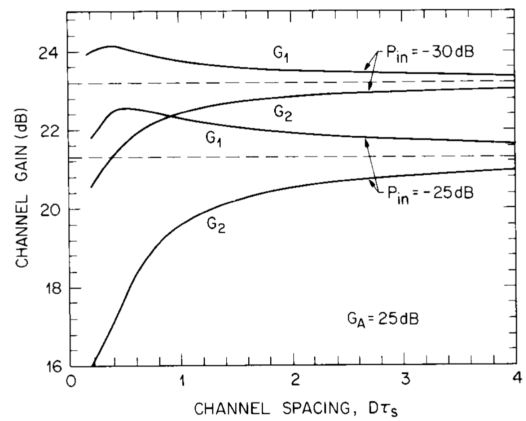

Fig. 1. Variation of channel gains G_1 and G_2 with channel spacing D for the case of a two-channel amplifier. In the absence of FWM, channel gains are equal and independent of the channel spacing (dashed line).

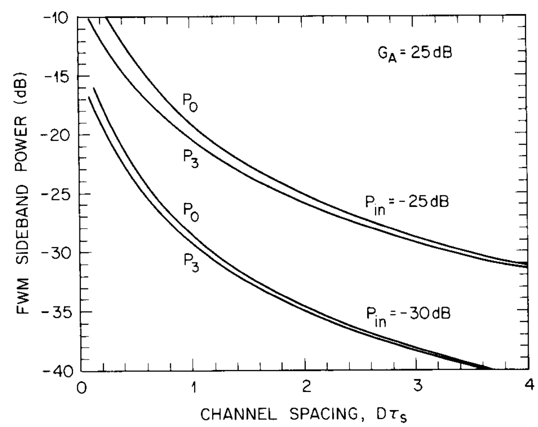

Fig. 2. Variation of the powers of the two sidebands generated by FWM with the channel spacing D. P_0 and P_3 correspond to low-frequency and high-frequency sidebands, respectively.

increases rapidly with an increase in the input power P_{in}. For relatively large values of P_{in} and small values of $D\tau_s$, the sideband can carry as much power as the channels themselves. This is the regime of strong FWM interaction that must be avoided if the objective is to reduce the interchannel crosstalk during multichannel amplification.

IV. Interchannel Crosstalk

The general result (12) can be used to consider simultaneous amplification of an arbitrary number of channels. To demonstrate its usefulness, we consider the specific case in which 30 channels are amplified simultaneously by the semiconductor laser amplifier ($M = 30$). We assume that the input power is the same for all channels. The input phase is generally not fixed and can vary randomly with time. For this reason we have averaged over the input phases in obtaining the following results. To quantify the effects of interchannel crosstalk, we consider

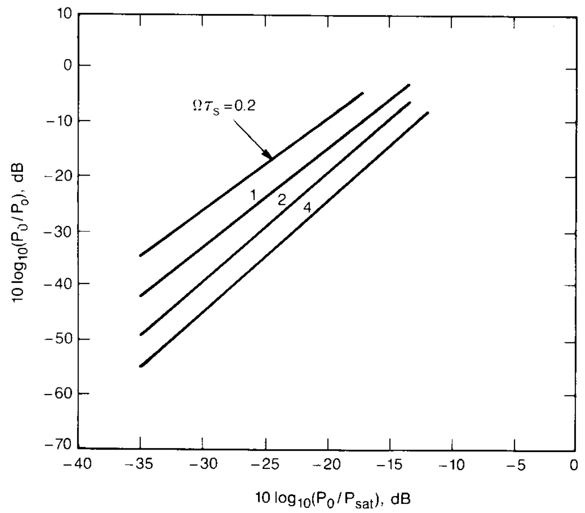

Fig. 3. Variation of P_i/P_0 with P_0 in the case of 30 channel amplification for several values of the normalized channel spacing $\Omega\tau_s$. P_0 is the channel output power in the absence of nonlinear effects and P_i is the power generated in the 15th channel as a result of amplification of all neighboring channels.

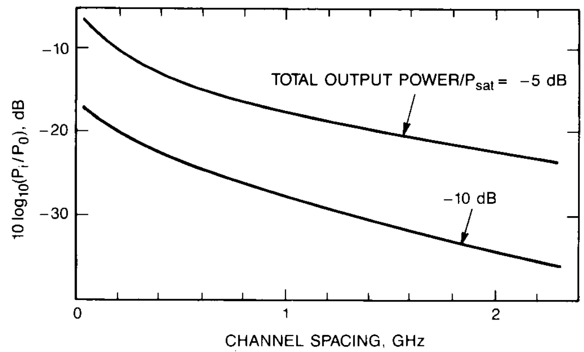

Fig. 4. Variation of P_i/P_0 with the channel spacing in the case of 30 channel amplification for two values of the total output power.

a specific middle channel (channel 15) and obtain the intermodulation power P_i generated in this channel as a result of FWM. More specifically, P_i is obtained by solving (12) with the input condition that $P_j = P_{in}$ for all channels except for channel 15 for which $P_{15} = 0$. The other parameters are identical to those used for Fig. 1.

Fig. 3 shows the intermodulation power P_i as a function of the output power P_0 of a channel expected in the absence of interchannel crosstalk. For convenience, the intermodulation power P_i is normalized to P_0; the ratio P_i/P_0 is a measure of intermodulation distortion occurring as a result of FWM in the amplifier. The different curves in Fig. 3 are for four different values of the parameter $\Omega\tau_s$, a measure of the channel spacing. As one would expect, intermodulation distortion increases when the channel spacing decreases. It also increases considerably when the amplifier is operated to obtain large channel powers at its output. Interchannel crosstalk sets an upper limit on the amplified channel power P_0. For example, if the design criterion is $P_i/P_0 < -20$ dB, then $P_0 < -22$ dB for $\Omega\tau_s = 1$ and becomes $P_0 < -18$ dB for $\Omega\tau_s = 4$.

A practical way to characterize the amplifier performance is to study the variation of P_i/P_0 with the channel spacing when the amplifier is operated to obtain a certain total output power P_{tot}. This is shown in Fig. 4 for $P_{tot} = -5$ and -10 dB (all powers are normalized to the saturation power of the amplifier) after assuming $\tau_s = 0.3$ ns. Intermodulation distortion decreases considerably (by more than 10 dB) when P_{tot} is changed from -5 to -10 dB. It also decreases with an increase in the channel spacing. For a channel spacing of 1 GHz, $P_i/P_0 = -17$ dB when $P_{tot} = -5$ dB and reduces to -27 dB when $P_{tot} = -10$ dB. Clearly the total output power has to be kept well below the saturation level in order to minimize the effects of FWM occurring as a result of carrier-density modulation.

V. Conclusion

We have developed a general theory of multichannel amplification in semiconductor laser amplifiers. The theory is valid in the large-signal regime and includes the effects of both gain saturation and carrier-density modulation on the evolution of the optical field associated with each channel. The carrier-density modulation manifests as FWM which transfers power from each channel to the neighboring channels. Such FWM unequalizes the channel gains and, at the same time, generates new sidebands on both sides of the boundary channels. We have discussed these effects by considering the cases of 2 and 30 channels. The theory presented here is expected to be useful in quantifying the amplifier-induced crosstalk levels in multichannel communication systems and indicates the operating conditions under which the crosstalk can be reduced to an acceptable level.

References

[1] For a review see, for example, M. J. O'Mahony, "Semiconductor laser optical amplifiers for use in future fiber systems," *J. Lightwave Technol.*, vol. 6, pp. 531-544, Apr. 1988.

[2] G. Grosskopf, R. Ludwig, and H. G. Weber, "Crosstalk in optical amplifiers for two-channel transmission," *Electron. Lett.*, vol. 22, pp. 900-902, 1986.

[3] T. Mukai, K. Inoue, and T. Saitoh, "Signal gain saturation in two-channel common amplification using a 1.5 μm InGaAsP travelling-wave laser amplifier," *Electron. Lett.*, vol. 23, no. 8, pp. 396-397, Apr. 9, 1987.

[4] R. M. Jopson, K. L. Hall, G. Eisenstein, G. Raybon, and M. S. Whalen, "Observation of two-colour gain saturation in an optical amplifier," *Electron. Lett.*, vol. 23, no. 10, pp. 510-512, May 7, 1987.

[5] M. G. Oberg and N. A. Olsson, "Crosstalk between intensity-modulated wavelength-division multiplexed signals in a semiconductor laser amplifier," *IEEE J. Quantum Electron.*, vol. QE-24, pp. 52-59, Jan. 1988.

[6] G. P. Agrawal, "Four-wave mixing and phase conjugation in semiconductor laser media," *Opt. Lett.*, vol. 12, no. 4, pp. 260-262, Apr. 1987.

[7] ——, "Amplifier-induced crosstalk in multichannel coherent light-

wave systems," *Electron. Lett.*, vol. 23, no. 22, pp. 1175–1177, Oct. 22, 1987.

[8] K. Inoue, T. Mukai, and T. Saitoh, "Nearly degenerate four-wave mixing in a traveling-wave semiconductor laser amplifier," *Appl. Phys. Lett.*, vol. 51, no. 14, pp. 1051–1053, Oct. 5, 1987.

[9] K. Inoue, "Observation of crosstalk due to four-wave mixing in a laser amplifier for FDM transmission," *Electron. Lett.*, vol. 23, no. 24, pp. 1293–1295, Nov. 19, 1987.

[10] T. E. Darcie, R. M. Jopson, and R. W. Tkach, "Intermodulation distortion in optical amplifiers from carrier-density modulation," *Electron. Lett.*, vol. 23, no. 25, pp. 1392–1394, Dec. 3, 1987.

[11] G. Grosskopf, R. Ludwig, R. G. Waarts, and H. G. Weber, "Four-wave mixing in a semiconductor laser amplifier," *Electron. Lett.*, vol. 24, no. 1, pp. 31–32, Jan. 7, 1988.

[12] G. P. Agrawal, "Population pulsations and nondegenerate four-wave mixing in semiconductor lasers and amplifiers," *J. Opt. Soc. Amer. B*, vol. 5, no. 1, pp. 147–159, Jan. 1988.

[13] T. E. Darcie and R. M. Jopson, "Nonlinear interactions in optical amplifiers for multifrequency lightwave systems," *Electron Lett.*, vol. 24, no. 10, pp. 638-640, May 12, 1988.

[14] A. A. M. Saleh, "Nonlinear models of travelling-wave optical amplifiers," *Electron. Lett.*, vol. 24, no. 14, pp. 835-837, July 7, 1988.

[15] I. M. I. Habbab and G. P. Agrawal, "Asymmetric channel gain and crosstalk in traveling wave optical amplifiers," *J. Lightwave Technol.*, vol. 7, no. 9, pp. 1351–1359, Sept. 1989.

[16] G. P. Agrawal and N. K. Dutta, *Long-Wavelength Semiconductor Lasers.* New York: Van Nostrand Reinhold, 1986, ch. 2.

Govind P. Agrawal (M'83–SM'86) was born in Kashipur, India. He received the M.S. and Ph.D. degrees from the Indian Institute of Technology, New Delhi in 1971 and 1974, respectively.

After spending several years at the Ecole Polytechnique, Palaiseau, France, and the City University of New York, New York, he became a member of the technical staff at AT&T Bell Laboratories. He joined the faculty of the Institute of Optics at the University of Rochester, Rochester, NY, in January 1989. His research interests focus on quantum electronics, nonlinear optics, and laser physics. In particular, he has contributed significantly to the fields of semiconductor lasers, nonlinear fiber optics, and optical communications. He is an author or coauthor of more than 150 research papers, several review articles, and two books entitled *Long-Wavelength Semiconductor Lasers* (Van Nostrand Reinhold Co., 1986) and *Nonlinear Fiber Optics* (Academic, 1989).

Dr. Agrawal is a Fellow of the Optical Society of America and a member of the American Physical Society.

Isam M. I. Habbab (M'86), photograph and biography not available at the time of publication.