Agrawal Replies: Rothenberg raises an important issue in his Comment.[1] The coupled nonlinear Schrödinger equations (NLSE) used in my paper[2] (referred to as the incoherently coupled equations or ICE's in his Comment) indeed neglect the four-wave interaction between the two input waves by assuming such interaction not be phase matched. His Comment, however, gives the impression that ICE's should never be used because of the neglect of the four-wave mixing or the coherent coupling term. He suggests that the coherent coupling term can be included by solving a single NLSE with an appropriate input. I take this opportunity to point out that this later approach has a limited validity in practice. Furthermore, the use of ICE's is valid in many cases where the spectrum of each wave remains narrower than the initial frequency separation between the two waves.[3]

Consider first the approach in which a single NLSE is used to describe the two-wave interaction by taking an input of the form

$$A(z,t) = A_1(z,t) + A_2(z,t) \exp[i(\Delta k z - \Delta \omega t)]$$

where $\Delta k = k_2 - k_1$, $\Delta \omega = \omega_2 - \omega_1$, and ω_1 and ω_2 are the optical frequencies of the two waves. Such an approach is valid for small frequency differences such that $\Delta \omega \ll \omega_{1,2}$. The reason is that the NLSE is derived from Maxwell's wave equation by assuming that $A(z,t)$ is a slowly varying function of time on a time scale of the optical period. In the example chosen by Rothenberg this condition is satisfied as the frequency difference of 2.6 THz is relative small. However, it will not be satisfied when the frequency difference exceeds 10 THz. In practical terms, this would be the case for waves whose wavelengths differ by as little as 10 nm. The main advantage of ICE's is that they are valid for arbitrary values of the wavelength difference between the two waves. Indeed, one wave can be in the infrared while the other is in the visible region, a situation not uncommon for experiments with optical fibers.

Having suggested that the use of ICE's is often dictated from practical considerations, I hasten to add that one must be careful to reject the solutions that violate the assumptions inherent in the derivation of ICE's. Rothenberg is thus right in pointing out that the linear-stability analysis of ICE's is likely to give wrong results if the modulation frequency Ω becomes comparable to $\Delta \omega$. This can happen for the case of a large group-velocity mismatch δ between the two waves. The question then becomes how large δ can be before ICE's break down for the discussion of modulation instability. An order-of-magnitude estimate can be obtained by introducing the concept of the walk-off length defined as the length by which the two waves walk off from each other over one modulation period $T_m = 2\pi/\Omega$. The walk-off length is related to δ and Ω by the relation $L_W = 2\pi/\Omega\delta$. One can also introduce a nonlinear length[3] defined as $L_{NL} = 1/\gamma P$, where γ is the nonlinearity parameter and P is the power assumed to be the same for both waves. The walk-off effects are negligible if $L_{NL} \ll L_W$ or $\delta \ll 2\pi\gamma P/\Omega$. It should be stressed that this condition is relevant in the context of modulation instability only. The general condition for the validity of ICE's is that four-wave mixing between the two copropagating waves does not occur throughout the nonlinear interaction. This condition is often satisfied in practice and the ICE's provide a proper framework to discuss the nonlinear phenomena.[3-5]

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